Minimizing Airplane Boarding Time

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Joint work with
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Agenda

The Airplane Boarding Problem I

How can the passenger boarding process be organized optimally?

Optimality depends on the **goal** to be considered, e.g.,

- maximize overall/average passenger satisfaction,
- maximize the minimum passenger satisfaction,
- minimize the number of contacts due to COVID-19,
- many more . . .

This talk: **Minimization of the overall boarding time.**

The Airplane Boarding Problem II

Why minimization of the overall boarding time?

- **Bottleneck** in total turnaround time of passenger airplanes.
- Cost savings estimations: $30–$250 per minute reduced turnaround time.

  ⇒ Major airlines with 5000 flights per day could save **$50M. per year**!

[Jaehn, Neumann, 2015]

All estimates are from before the COVID-19 pandemic.

The Airplane Boarding Problem III

How can we minimize the overall airplane boarding time?

Organize/Modify the passenger order before entering the airplane cabin.

- By group: Call (predefined) groups at the gate. Order within groups is "random".

  Examples: Back-to-Front, Outside-In, Reverse-Pyramid, . . .

- By seat: Predefine an order of all passengers by passenger attributes.

  Examples: Steffen Method, Outside-In with Back-to-Front within groups, . . .

- Classical boarding strategies rely solely on the predefined seats of the passengers.

This talk: **Use more passenger-specific data to optimize the boarding order.**

Mathematical Problem Definition

Cabin Layout:

- only one aisle and door/entry — right in front or at the back of the cabin
- rows $\mathcal{R}$ — $= (1, 2, \ldots, R)$
- seats $\mathcal{S}$ — $\mathcal{S}_r^1 := ((r,1), \ldots, (r, k_r^1)), \mathcal{S}_r^2 := ((r, k_r^1 + 1), \ldots, (r, k_r^1 + k_r^2))$

Passengers

- passengers $\mathcal{P}$ — $= \{1, 2, \ldots, P\}$
- predefined seat $(r(p), s(p))$
- moving times t_{pr}^m — $\in \mathbb{Q}_{\geq 0}$ for passing a row $r \leq r(p) - 1$
- settling-in times t_p^s — $\in \mathbb{Q}_{\geq 0}$ for all actions in row $r(p)$ esp. stowing away luggage

Goal:

- Find a **boarding order** $\pi : \mathcal{P} \to \{1, 2, \ldots, P\}$ minimizing total boarding completion time.

Assumptions

General:

- Every passenger tries to move directly to their seat.
- Luggage is stowed away at the row of the resp. assigned seat.
- A passenger in row r blocks the complete row.
- Overtaking in the aisle is not allowed/possible.

For now (reconsidered later)**:**

- Complete access to the passenger-specific data.
- No influence of seat interferences.
- No passenger groups, e.g., families, who want to board together.

Agenda

Intractability

Availability of passenger-specific moving and settling-in time data pertains to an ideal situation that would allow for the most considerate boarding sequence planning.

Theorem [WT'19]: The Airplane Boarding Problem (ABP) is NP-hard in the strong sense.

This holds true for very simple cabin layouts, e.g.,

- only one seat per row, all-zero settling-in times, and integral moving times
- only two rows, all-zero moving times, and integral settling-in times

Corollary [WT'19]: There is no polynomial-time exact, no pseudo-polynomial exact, and no fully polynomial-time approximation algorithm, unless $\mathrm{P} = \mathrm{NP}$.

Weakening the Ideal Data Assumption

- Simulation data can be as detailed as desired.
- In practice, one will hardly have detailed moving and settling-in time information on every passenger, and will thus have to use estimates.

Lemma [WT'19]: Given an optimal ABP solution with completion time $C_{\max}$, decreasing some moving and settling-in time values cannot increase the optimal completion time.

- Justification for using "pessimistic" estimations.
- Provides a certain notion of robustness.

Simplifications

It can make sense to restrict to

(1) same number of seats per row $|\mathcal{S}_r^1| + |\mathcal{S}_r^2| = k \ \forall\, r \in \mathcal{R}$

(2) identical moving times $t_{p,r}^m \equiv t^m \ \forall\, p \in \mathcal{P},\ r \in \mathcal{R}$

(3) identical settling-in times $t_p^s \equiv t^s \ \forall\, p \in \mathcal{P}$

- Only one restriction, ABP remains strongly NP-hard.
- Under (1), (2), and (3), ABP becomes easy, i.e., polynomial time solvable.
- Under (1) and (2), we can prove approximation guarantees.

Agenda

1. Introduction to the Airplane Boarding Problem
2. Computational Complexity of Airplane Boarding
3. **Heuristics and Approximation Algorithms**
4. Exact Mixed-Integer Programming Approach & Extensions
5. Computational Experiments

Outside-In Boarding Strategy

Outside-In (by seat):

- Window-to-Aisle and Back-to-Front (within groups)
- performs in many simulations among the best
- closed-form expression for position of passenger

	Front						
1	20	60	100		120	80	40
2	19	59	99		119	79	39
3	18	58	98		118	78	38
4	17	57	97		117	77	37
5	16	56	96		116	76	36
6	15	55	95		115	75	35
7	14	54	94		114	74	34
8	13	53	93		113	73	33
9	12	52	92		112	72	32
10	11	51	91		111	71	31
11	10	50	90		110	70	30
12	9	49	89		109	69	29
13	8	48	88		108	68	28
14	7	47	87		107	67	27
15	6	46	86		106	66	26
16	5	45	85		105	65	25
17	4	44	84		104	64	24
18	3	43	83		103	63	23
19	2	42	82		102	62	22
20	1	41	81		101	61	21

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Polynomially-Solvable Special Case

Theorem [WT'19]: Outside-In is optimal for restricted ABP instances:

- fully booked airplane $|\mathcal{P}| = |\mathcal{S}|$
- constant number of seats per row $|\mathcal{S}_r^1| + |\mathcal{S}_r^2| \equiv k \in \mathbb{Z}_{\geq 0}$
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$$\max_{r \in \mathcal{R}} \left(\min_{p \in \mathcal{P}: r(p) \geq r} \sum_{r'=1}^{r-1} t_{p,r'}^m + \sum_{p \in \mathcal{P}: r(p) \geq r+1} t_{p,r}^m + \sum_{p \in \mathcal{P}: r(p) = r} t_p^s \right)$$

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- First passenger p blocking row r has to pass rows $1, \ldots, r-1$.

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- All passenger p with seat in row $r(p) \geq r+1$ must pass row r.

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- All passenger p with seat in row $r(p) = r$ have to settle in.

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- Since the previous holds for all rows $r \in \mathcal{R}$, we can maximize over them.

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- In current setting with $r = 1$, the lower bound collapses to $k \cdot t^s + k \cdot (R-1) \cdot t^m$ $\square$

Approximation Algorithm I (Outside-In)

Approximation Algorithm: An α-approximation algorithm $\mathcal{A}$ for ABP is a polynomial time algorithm that produces for every instance $\mathcal{I}$ of (restricted) ABP a solution with

$$\mathcal{A}(\mathcal{I}) \leq \alpha \cdot \mathrm{OPT}(\mathcal{I}).$$

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Outside-In Asymptotically

Theorem [WT'19]: Outside-In is asymptotically optimal for restricted ABP instances:

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- constant moving times for all passengers $t_{p,r}^m \equiv t^m \in \mathbb{Q}_{\geq 0}$
- bounded but arbitrary settling-in times $t_p^s \in (0, B], B < \infty$
- number of rows tends to infinity $|\mathcal{R}| \to \infty$

Well-known boarding strategies are inherently flawed

Theorem [WT'19]: For every $R, k \geq 2$, there exists an ABP instance for which every deterministic boarding strategy that disregards passengers' individual settling-in times leads to a boarding time at least **twice as long as** the **optimal** boarding time.

Theorem [W'20]: For every $R \geq 3$ and $k \geq 5$, there exists an ABP instance for which every deterministic boarding strategy that disregards passengers' individual settling-in times leads to a boarding time at least **three times as long as** the **optimal** boarding time.

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Problem: Classical boarding strategies (all?) rely solely on seat assignments, including

Outside-In, Back-to-Front, Reverse Pyramid, Steffen Method, ...

Outside-In: There are instances with boarding time factor $2k - \varepsilon$ worse than optimal.

Approximation Algorithm II (Max-Settle-Row)

Max-Settle-Row:

- Modification of Outside-In: Back-to-Front regime remains, but not Window-to-Aisle.
- First, passenger from last row, then passenger from the second last row, etc..
- Passengers of a row are non-increasingly sorted by their settling-in time.

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Theorem [WT'19]: Max-Settle-Row is $(1 + \frac{H_k(k-1)}{k})$-approximation for restr. ABP instances:

- fully booked airplane and constant number of seats per row
- constant moving times for all passengers $t^m_{p,r} \equiv t^m \in \mathbb{Q}_{\geq 0}$
- arbitrary settling-in times $t^s_p \in \mathbb{Q}_{\geq 0}$

$H_k = \sum_{\ell=1}^{k} \frac{1}{\ell}$, $(1 + \frac{H_k(k-1)}{k}) < k \; \forall \, k \geq 2$, $k = 6$: $1 + \frac{5H_6}{6} = 1 + \frac{12.25}{6} \approx 3.042 < 6$

$\Rightarrow$ Max-Settle-Row inherits all perks of Outside-In and gives an improved approximation bound.

Agenda

A Compact MIP Formulation (Variables and Parameters)

- Assignment variables $x_{pi} \in \{0, 1\}$ with $x_{pi} = 1 \Leftrightarrow \pi(p) = i$.
- Time of arrival / finishing variables t^A_{ir}, $t^F_{ir} \in \mathbb{Q}_{\geq 0}$ for the i-th passenger and row $r \in \mathcal{R}$.
- Overall boarding completion time variable $C_{\max} \in \mathbb{Q}_{\geq 0}$.
- Let τ_{pr} be the time data passenger $p \in \mathcal{P}$ must spend in row $r \in \mathcal{R}$

$$\tau_{pr} = \begin{cases} t^m_{pr}, & r < r(p) \\ t^s_p, & r = r(p) \\ 0, & r > r(p) \end{cases}$$

passing row r

settling-in at row r

auxiliary, each passenger virtually goes to last row

A Compact MIP Formulation (Constraints)

- Assignment constraints:

$$\sum_{p \in \mathcal{P}} x_{pi} = 1 \quad \forall\, i \in [P], \qquad \sum_{i \in [P]} x_{pi} = 1 \quad \forall\, p \in \mathcal{P}$$

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- Time related constraints:

$$\sum_{p\in\mathcal{P}} \tau_{pr} x_{pi} \leq t^F_{ir} - t^A_{ir} \qquad \forall\, i,\ \forall\, r \qquad \text{spend time } \geq \tau_{pr} \text{ in row } r$$

Coupling the times a passenger needs in row r with the times of their position.

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- Time related constraints:

$$\sum_{p\in\mathcal{P}} \tau_{pr} x_{pi} \leq t^F_{ir} - t^A_{ir} \qquad \forall\, i,\ \forall\, r \qquad \text{spend time } {\geq}\tau_{pr} \text{ in row } r$$

$$t^F_{i,r-1} \leq t^A_{ir} \qquad \forall\, i,\ \forall\, r \geq 2 \qquad \text{process rows consecutively}$$

Time starting in row r has to be at least the time finishing row $r-1$.

A Compact MIP Formulation (Constraints)

- Assignment constraints:

$$\sum_{p \in \mathcal{P}} x_{pi} = 1 \quad \forall\, i \in [P], \qquad \sum_{i \in [P]} x_{pi} = 1 \quad \forall\, p \in \mathcal{P}$$

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$$t^F_{i,r-1} \leq t^A_{ir} \qquad \forall\, i,\ \forall\, r \geq 2 \qquad \text{process rows consecutively}$$

$$t^F_{i-1,r} \leq t^A_{ir} \qquad \forall\, i \geq 2,\ \forall\, r \qquad \text{overtaking not possible}$$

Overtaking is not allowed/possible.

A Compact MIP Formulation (Constraints)

- Assignment constraints:

$$\sum_{p \in \mathcal{P}} x_{pi} = 1 \quad \forall\, i \in [P], \qquad \sum_{i \in [P]} x_{pi} = 1 \quad \forall\, p \in \mathcal{P}$$

- Time related constraints:

$\sum_{p \in \mathcal{P}} \tau_{pr} x_{pi} \leq t^F_{ir} - t^A_{ir}$	$\forall\, i,\ \forall\, r$	spend time $\geq \tau_{pr}$ in row r
$t^F_{i,r-1} \leq t^A_{ir}$	$\forall\, i,\ \forall\, r \geq 2$	process rows consecutively
$t^F_{i-1,r} \leq t^A_{ir}$	$\forall\, i \geq 2,\ \forall\, r$	overtaking not possible
$t^A_{ir} - t^F_{i,r-1} \leq \sum_{p \in \mathcal{P}: r(p) \leq r-1} M \cdot x_{pi}$	$\forall\, i,\ \forall\, r \geq 2$	time arriving in next row

Starting in row r directly after finishing in row $r-1$ for $r \leq r(p)$ and
i-th passenger does not block any, if virtually going to the end $(r > r(p))$

A Compact MIP Formulation

$$\min \quad C_{\max} \qquad \text{minimize boarding time}$$

$$\text{s.t.} \quad \sum_{p\in\mathcal{P}} x_{p,i} = 1 \quad \forall\, i \in \mathcal{P}, \quad \sum_{i\in[P]} x_{pi} = 1 \quad \forall\, p \in \mathcal{P} \qquad \text{permutation constraints}$$

$$C_{\max} \geq t^F_{iR} \qquad \forall\, i \qquad \text{couple completion time}$$

$$t^F_{ir} - t^A_{ir} \geq \sum_{p\in\mathcal{P}} \tau_{pr} x_{pi} \qquad \forall\, i, \forall\, r \qquad \text{spend time } \geq \tau_{pr} \text{ in row } r$$

$$t^A_{ir} \geq t^F_{i,r-1} \qquad \forall\, i \geq 2,\ \forall\, r \qquad \text{process rows consecutively}$$

$$t^A_{ir} \geq t^F_{i-1,r} \qquad \forall\, i \geq 2,\ \forall\, r \qquad \text{overtaking not possible}$$

$$M \cdot \sum_{p\,\in\,\mathcal{P}:r(p)\leq r-1} x_{pi} \geq t^A_{ir} - t^F_{i,r-1} \qquad \forall\, i,\ \forall\, r \geq 2 \qquad \text{time arriving in next row}$$

$$t^A,\ t^F \in \mathbb{Q}^{[P]\times\mathcal{R}}_{\geq 0},\ C_{\max} \in \mathbb{Q}_{\geq 0},\ x \in \{0,1\}^{\mathcal{P}\times[P]} \qquad \text{variables}$$

Extensions I (Seat Interferences)

- Prohibit seat interferences altogether by

$$1 - x_{p_1,i_1} \geq x_{p_2,i_2}$$

$$\forall\,(i_1,i_2) \in [P]^2 : i_1 > i_2,\ \forall\,(p_1,p_2) \in \mathcal{P}^2_{\neq},\ r(p_1) = r(p_2),\ s(p_1) < s(p_2) \leq k^1_{r(p_1)}$$

analogously for other side of the aisle

Extensions I (Seat Interferences)

- Prohibit seat interferences altogether by

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- Let $\eta_p \in \mathbb{Q}_{\geq 0}$ be the time data $p \in \mathcal{P}$ needs to let someone pass and $z_{pi} \in \mathbb{Q}_{\geq 0}$ be a variable to capture additional waiting times.

$$M_r(x_{pi} - 1) + \sum_{\substack{p' \in \mathcal{P}(r) \\ s(p') < s(p)}} \sum_{i' < i} \eta_{p'} x_{p',i'} \leq z_{pi} \quad \forall\, i \in [P],\, r \in \mathcal{R},\, p \in \mathcal{P}(r) : s(p) < k^1_r$$

and change $t^F_{ir} - t^A_{ir} \geq \sum_{p \in \mathcal{P}} (\tau_{pr} x_{pi} + \mathbf{1}_{r=r(p)} z_{i,p})\ \forall i \in \mathcal{P}, r \in \mathcal{R}$

analogously for other side of the aisle

Extensions II (Inseparable Passenger Groups)

- Supposing order of passengers within group is fixed a priori.
  Ensure ordered group $(p_1, \ldots, p_\ell) \subseteq \mathcal{P}$ sits together by

$$x_{p_{j+1},i+j+1} = x_{p_j,i+j} \quad \forall\, i \in \{0, 1, \ldots, P - \ell\},\ \forall\, j \in \{1, \ldots, \ell - 1\}.$$

Extensions II (Inseparable Passenger Groups)

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  Ensure ordered group $(p_1, \ldots, p_\ell) \subseteq \mathcal{P}$ sits together by

$$x_{p_{j+1},i+j+1} = x_{p_j,i+j} \quad \forall\, i \in \{0, 1, \ldots, P - \ell\},\ \forall\, j \in \{1, \ldots, \ell - 1\}.$$

- Supposing order of passengers within group can be arbitrary.

  Let $\mu_i^G \in \{0, 1\}$ variable indicating whether group G starts boarding at position $i + 1$.

$$\sum_{i=0}^{P-|G|} \mu_i^G = 1; \qquad \sum_{p\in G} \sum_{j=1}^{|G|} x_{p,i+j} \geq |G| \cdot \mu_i^G \quad \forall i \in \{0, \ldots, P - |G|\}$$

Adapting Heuristics to Extensions

Seat Interferences:

- Outside-In and Steffen Method avoid seat interferences by design.
- Enforcing seat interference avoidance in Max-Settle-Row results in Outside-In.
- Enforcing seat interference avoidance in Random yields randomized Outside-In.

Groups:

- Inseparable passenger groups can be taken into account for heuristics, e.g.,
  Schedule whole group as soon as one members is encountered.
  Order within group like strategy (e.g., Outside-In).
- Incorporating inseparable groups introduces seat interferences in strategies avoiding them.

Agenda

Test Setup

- 4 different cabin layouts with R rows and k seats per row:

$$(R, k) \in \{(10, 2), (20, 2), (20, 4), (30, 6)\}$$

- Passenger-seat assignment was chosen uniformly at random.
- All time data where chosen randomly, e.g., $t_{pr}^{m} \in \{1, 2, 3\}$ with probabilities $\{\frac{1}{4}, \frac{1}{2}, \frac{1}{4}\}$.
- 5 primal heuristics: Outside-In, Max-Settle-Row, and Steffen Method; Random Outside-In and Random both with 1000 trials.
- Improvement heuristic: simple local search (2-opt swaps)
- MIP provided with best (improved) heuristic solution.

Experimental Results I

method	objective	w/ 2-opt	% impr.	% gap	# opt	runtime
Random	1383.1	747.4	45.96	—	—	18.3
Steffen	778.2	660.8	15.08	—	—	13.8
Rand. Outside-In	1326.5	750.7	43.41	—	—	14.8
Outside-In	514.6	462.9	10.04	—	—	12.2
Max-Settle-Row	475.5	454.6	4.40	—	—	9.7
MIP	452.7	—	0.19	12.37	10	2629.3

Table: Results with average values over 40 instances, 10 each for 4 airplane sizes. MIP time limit: 1 h.

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Experimental Results I

method	objective	w/ 2-opt	% impr.	% gap	# opt	runtime
Random	1383.1	747.4	45.96	—	—	18.3
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Table: Results with average values over 40 instances, 10 each for 4 airplane sizes. MIP time limit: 1 h.

Random and Random Outside-In produce worst results.

Experimental Results I

method	objective	w/ 2-opt	% impr.	% gap	# opt	runtime
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Outside-In, Max-Settle-Row, and MIP produce the best results.

Experimental Results I

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Table: Results with average values over 40 instances, 10 each for 4 airplane sizes. MIP time limit: 1 h.

Improvement of the simple local search heuristic is remarkable, esp. for worse heuristics.

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Table: Results with average values over 40 instances, 10 each for 4 airplane sizes. MIP time limit: 1 h.

Empirical approximation guarantee via MIP lower bounds for Max-Settle-Row: 1.17.

Experimental Results II: Sensitivity & Robustness

method +2-opt	obj.	late	displ.	pert. obj.	comb.	ref. obj.	% comb. loss
Random	747.4	891.7	1000.5	833.1	1151.4	679.8	[60.7, 64.9]
Steffen	660.8	812.8	928.9	740.0	1077.2	644.6	[58.0, 62.5]
Rand. Outside-In	750.7	891.5	1002.4	838.1	1155.6	683.8	[60.9, 65.1]
Outside-In	462.9	615.5	726.7	513.4	888.2	467.7	[49.1, 54.5]
Max-Settle-Row	454.6	609.3	711.7	497.5	874.8	467.8	[48.3, 53.8]
MIP	452.7	607.4	711.0	500.5	880.5	464.1	[48.7, 54.1]

Table: Results of robustness experiments on test set mp-sp. Numbers are average values over 40 instances, 10 each for 4 airplane sizes. MIP time limit: 1 hour.

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Table: Results of robustness experiments on test set mp-sp. Numbers are average values over 40 instances, 10 each for 4 airplane sizes. MIP time limit: 1 hour.

Late and displaced passengers have a great influence on the overall boarding time.

Experimental Results II: Sensitivity & Robustness

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Random	747.4	891.7	1000.5	833.1	1151.4	679.8	[60.7, 64.9]
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Time perturbations are negligible compared to other disruption causes.

Experimental Results II: Sensitivity & Robustness

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Random	747.4	891.7	1000.5	833.1	1151.4	679.8	[60.7, 64.9]
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Combined disruptions most realistic scenario.

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Table: Results of robustness experiments on test set mp-sp. Numbers are average values over 40 instances, 10 each for 4 airplane sizes. MIP time limit: 1 hour.

Combined loss intervals: min. and max. worsening by disruption.

Experimental Results III: Groups

groups	method	objective	w/ 2-opt	% impr.	% gap	# opt	runtime
25%	Random	1951.4	1186.4	39.20	—	—	14.8
	Outside-In	1369.3	947.8	30.78	—	—	11.6
	MIP	929.6	—	1.88	35.63	11	2954.8
50%	Random	2028.2	1272.4	37.27	—	—	12.0
	Outside-In	1941.3	1129.3	41.83	—	—	11.5
	MIP	1089.9	—	3.11	43.14	10	1985.2
75%	Random	2058.5	1319.4	35.91	—	—	9.3
	Outside-In	2438.4	1239.8	49.15	—	—	9.0
	MIP	1187.2	—	2.85	47.34	10	1996.5

Table: Experimental results on testset mp-sp with inseparable groups. Numbers are average values over 40 instances, 10 each for 4 airplane sizes. MIP time limit: 1 hour.

Experimental Results III: Groups

groups	method	objective	w/ 2-opt	% impr.	% gap	# opt	runtime
25%	Random	1951.4	1186.4	39.20	—	—	14.8
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MIP outperforms Random and Outside-In.

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Table: Experimental results on testset mp-sp with inseparable groups. Numbers are average values over 40 instances, 10 each for 4 airplane sizes. MIP time limit: 1 hour.

Outside-In after local search almost as good as MIP.

Experimental Results III: Groups

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Table: Experimental results on testset mp-sp with inseparable groups. Numbers are average values over 40 instances, 10 each for 4 airplane sizes. MIP time limit: 1 hour.

Local search improves a lot.

Experimental Results III: Groups & Seat Interferences

groups	method	objective	w/ 2-opt	% impr.	% gap	# opt	runtime
25%	Random	2018.4	1200.0	40.55	—	—	21.0
	Outside-In	1369.3	953.6	30.36	—	—	15.8
	MIP	937.1	—	1.72	36.31	10	2913.3
50%	Random	2086.5	1262.8	39.48	—	—	17.3
	Outside-In	1942.0	1134.6	41.58	—	—	16.2
	MIP	1088.3	—	3.39	43.00	10	1900.3
75%	Random	2113.7	1342.0	36.51	—	—	13.4
	Outside-In	2440.2	1247.4	48.88	—	—	12.7
	MIP	1196.1	—	3.22	47.88	10	1923.6

Table: Experimental results on testset mp-sp with inseparable groups and seat interference consideration. Numbers are average values over 40 instances, 10 each for 4 airplane sizes. MIP time limit: 1 hour.

In this experiment: seat interference has no big influence on the overall boarding time.

Experimental Results IV: All Together

method	obj.	late	displ.	comb.	% comb. loss
Random + 2-opt	1268.2	1569.3	1432.2	1701.4	[36.9, 63.9]
Outside-In + 2-opt	1111.9	1418.2	1299.4	1565.2	[31.4, 60.8]
MIP	1073.8	1387.7	1267.1	1537.0	[30.1, 60.1]

Table: Results of robustness experiments on testset mp-sp with inseparable passenger groups and seat interferences. Considered were disruptions caused by late passengers, passenger displacements, and both combined. Numbers are average values over 120 instances; 30 each for 4 airplane sizes with 25%, 50%, and 75% of passengers in groups. MIP time limit: 1 hour.

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In the most realistic setting, Outside-In with local search is almost as good as MIP.

Conclusions & Outlook

- Boarding time can can be significantly improved by considering passengers time data.
- Local search drastically improves heuristic solutions;
  "Max-Settle-Row + 2-opt" often close optimal.
- MIP enables quantitative (empirical) statements about heuristic accuracy.
- MIP "black-box" may be refined with problem-specific components.
- How can by-seat strategies implemented in practice?
- Boarding is highly sensitive to disruptions.
  ⇒ Our future work: robust optimization to safeguard against disruptions.

Thanks!

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