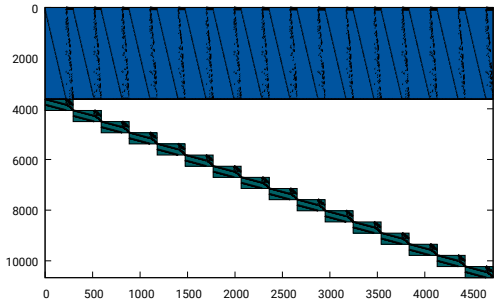
Modular Detection of Model Structure in Integer Programming

Michael Bastubbe & Marco Lübbecke

SCIP Workshop 2018
March 8th, 2018

Brief overview: Dantzig-Wolfe Reformulation

- Partition constraints of a MILP:
  - Master problem $Ax \geq b$
  - Pricing problems $D_i x_i \geq d_i$



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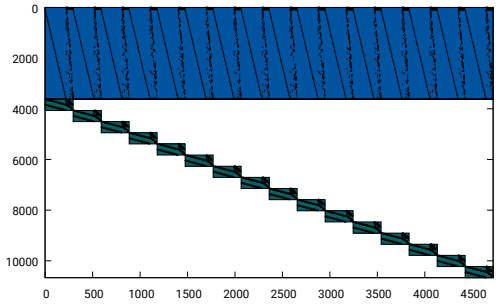

- Reformulate master problem: combination of extreme points and rays ("convexification") of pricing problems
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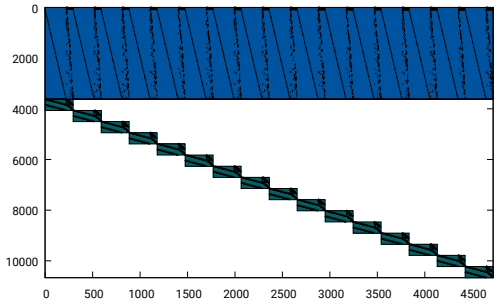
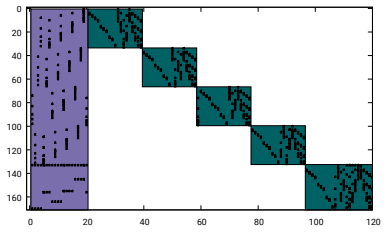


- Reformulate master problem: combination of extreme points and rays ("convexification") of pricing problems
- Solve master problem with Branch-Price-and-Cut
- Allow linking variables (each is copied for every linked block)

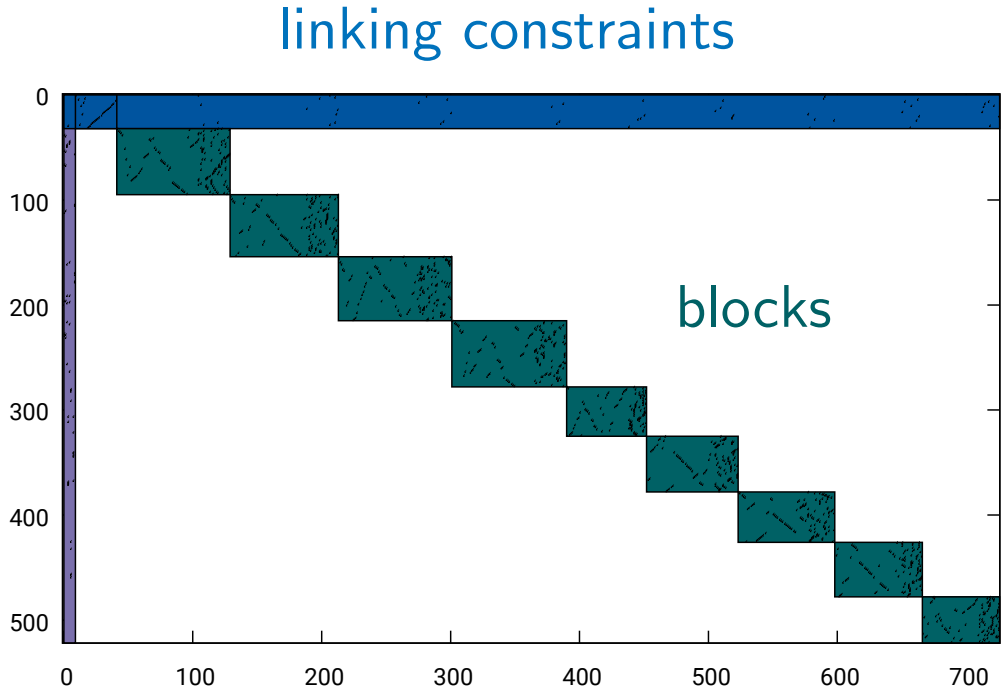


Outline: What is new?

- modular detection scheme
- constraint and variable classification
- consider information from original problem
- guess meaningful number of blocks
- pluggable score to evaluate decompositions
- user interaction

What are we looking for?

find:



Modular scheme

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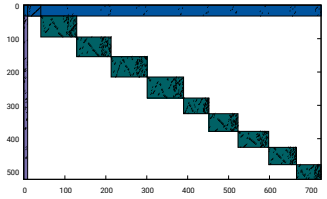

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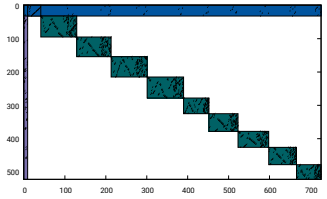

past:

- each detector is called once; returns set of decompositions
- no interaction between detectors
- “connected” detector implemented a mix of two ideas

Modular scheme

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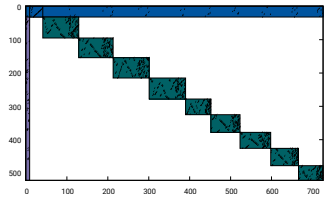


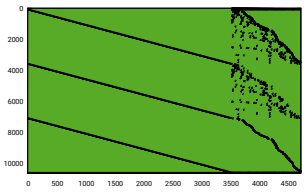
past:

- each detector is called once; returns set of decompositions
- no interaction between detectors
- "connected" detector implemented a mix of two ideas

recent development:

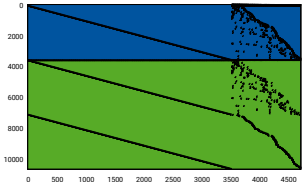
- each detector persues one atomic idea
- mixtures are handled in a general scheme
- new concept: partially decided decomposition [aka partials]
- partials are refined round wise

Example for partially refinement



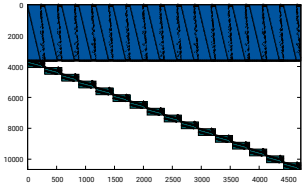
id 2; nB 0; maxW$\geq$ 0.00

consclass_constypes:setpartitioning-linear-logicor-knapsack



id 25; nB 0; maxW$\geq$ 0.34

connected



id 122; nB 16; maxW 0.38

Overview: What is new?

recent dev:

- new concept: partially decided decomposition [aka partials]
- partials are refined round wise
- detectors can implement three different callbacks:

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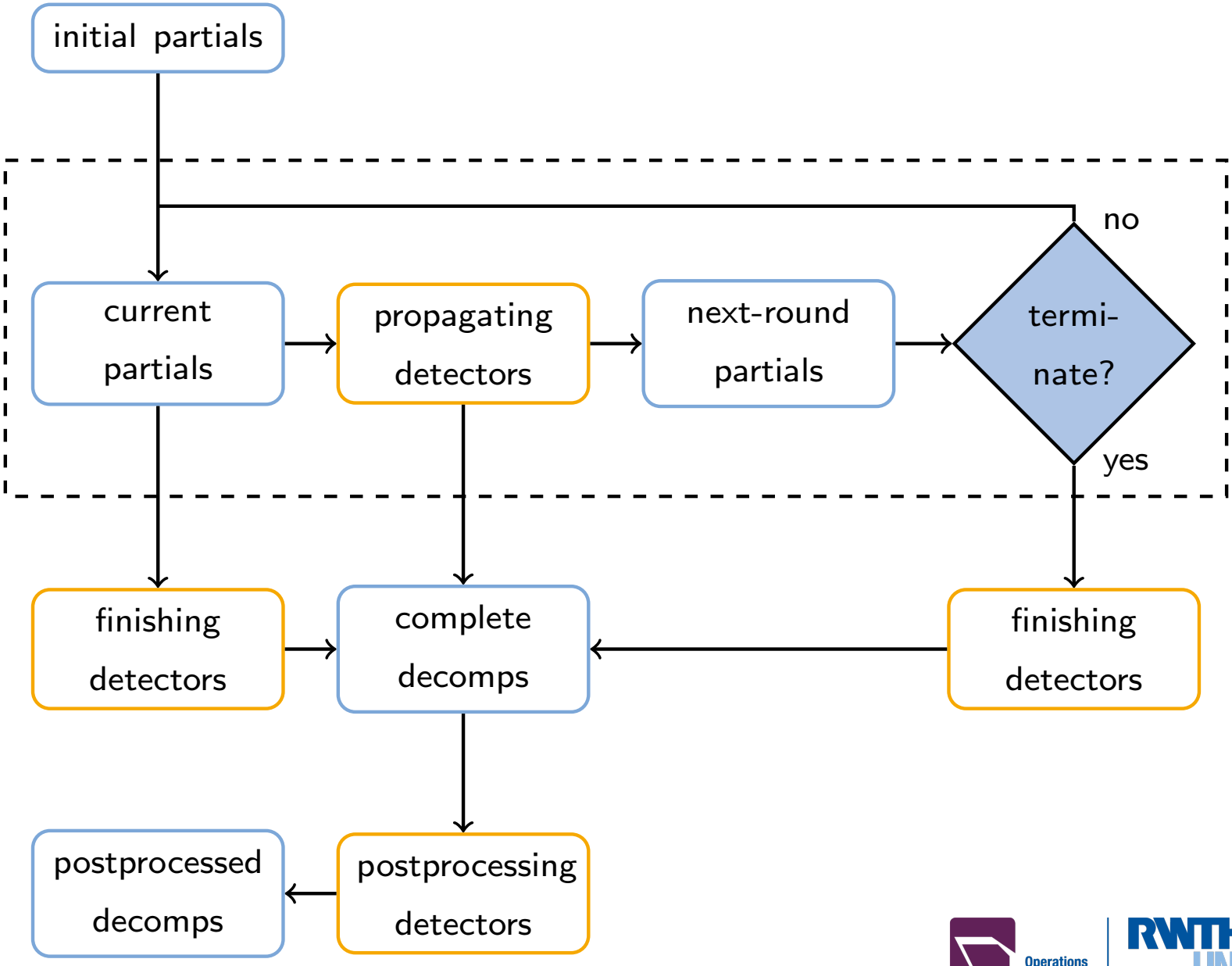
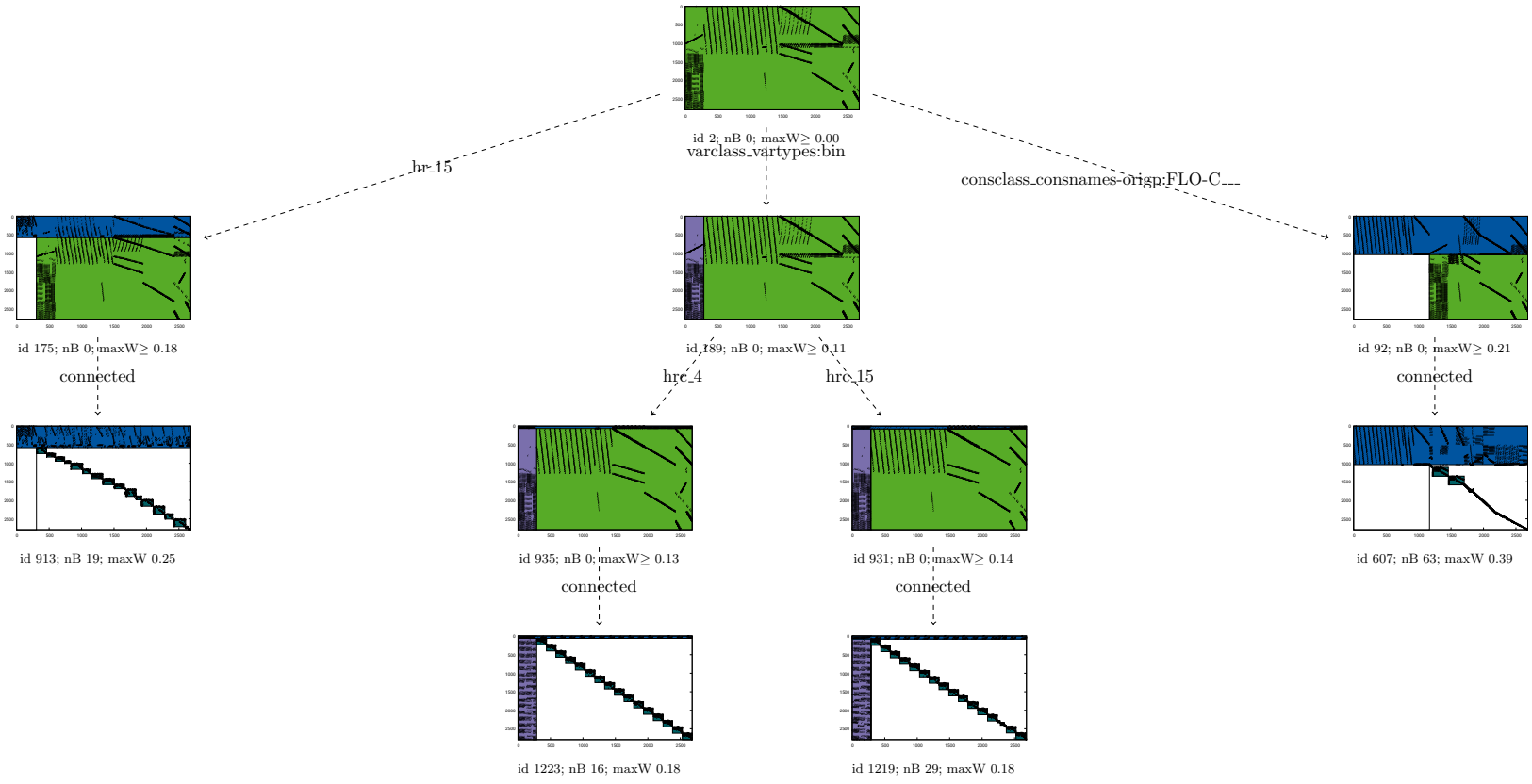
callback	input	output	revoke decisions?
propagate	partial	mix of complete/partials	no
finish	partial	set of complete	no
postprocess	complete	set of complete	yes

Detection overview: detection loop



Modular Detection: Example *b2c1s1*

- resulting decompositions can have common ancestors



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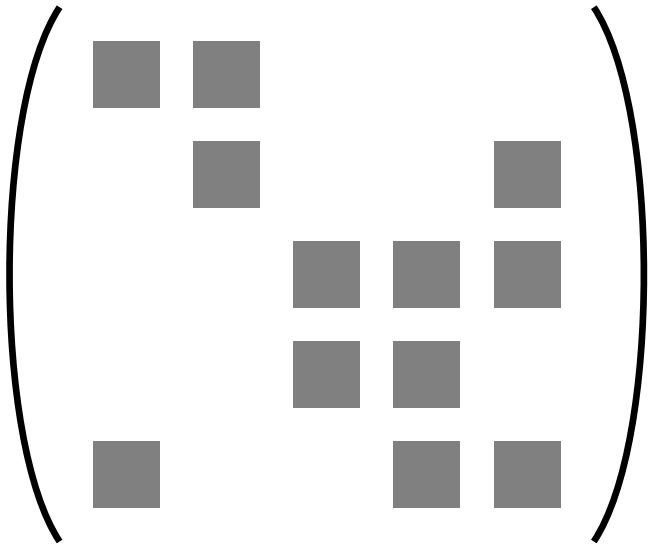
Example Propagating Detector: hrcgpartition

- Row-column-net hypergraph used in Bergner et al. (2015)
- Every nonzero entry a_{ij} is a vertex
- Every row and column is a hyperedge
- Solve balan. $\min$-k way cut problem
- Hyperedges between partitions in border



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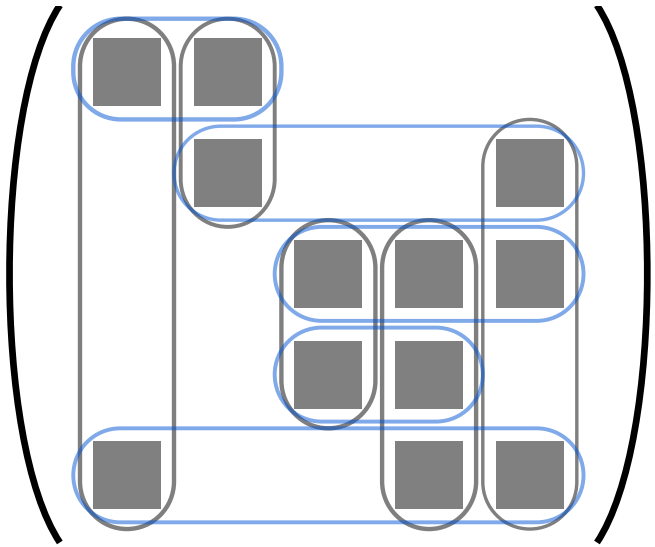

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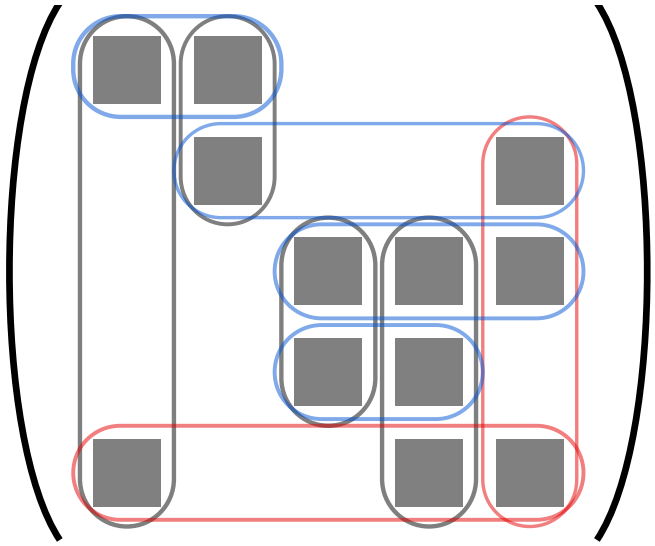

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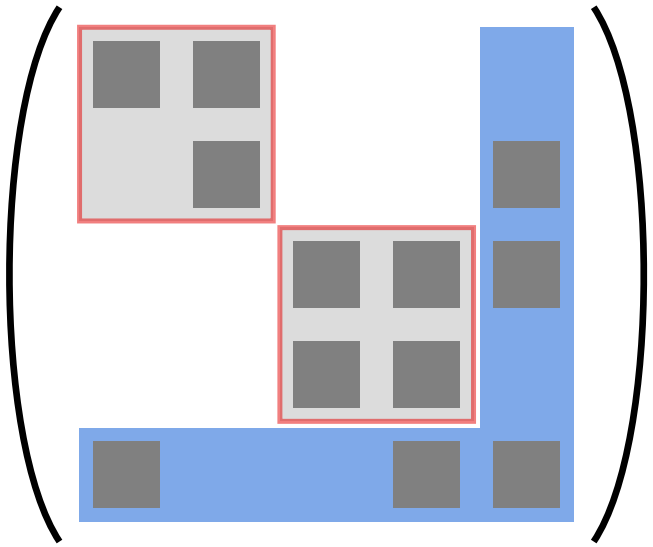

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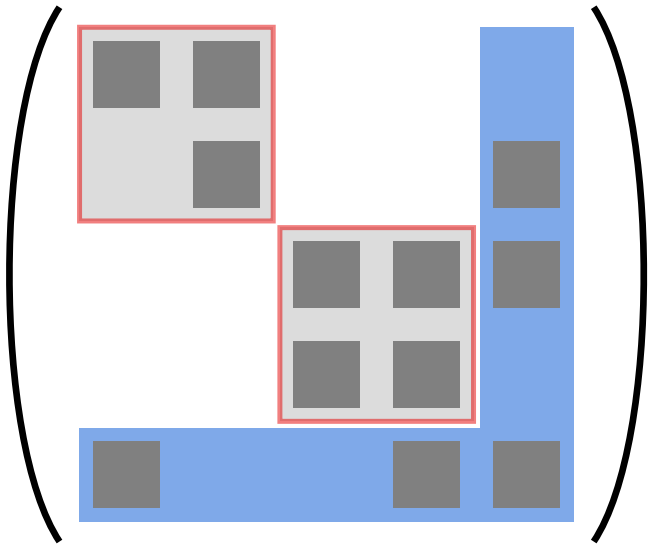
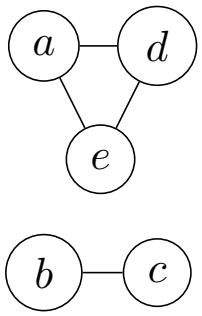


- Note:
  - returns complete and partial (only master and linking is assigned)
  - k is not known
  - problem is $\mathcal{NP}$-hard

Example finishing detector: Connected

- connected component in row-adjacency graph yields a block

$$\begin{array}{c} \\ a \\ b \\ c \\ d \\ e \end{array}\begin{array}{c} \begin{array}{ccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 \end{array} \\ \begin{pmatrix} 1 & & & & & & 1 \\ & 1 & & 1 & & 1 & \\ & & & 1 & 1 & 1 & \\ 1 & & 1 & & & & \\ & & 1 & & & & 1 \end{pmatrix} \end{array}$$



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Example postprocessing detector

- simple idea: check for each master constraint if it can be assigned to exactly one block

Constraint Classification: Binpacking example

$$
\begin{aligned}
\text{min } & y_1 + y_2 + y_3 \\
\text{s.t. } & -100y_1 + 54x_{11} + 33x_{21} + 34x_{31} + 72x_{41} \leq 0 \\
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- several classifications (aka classifier) possible
- during detection loop: a propagating detector assigns combinations of classes to the master for each classifier

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- several constraint classifiers:
  - same number of nonzero entries
  - type found by *SCIP*
  - type according to *MIPLIB2010*
  - names differ only by digits
  - ...

Variable classification

- Several variable classifiers:
  - type according to *SCIP*
  - objective function coefficient
  - sign of objective function coefficient
  - ...

Variable classification

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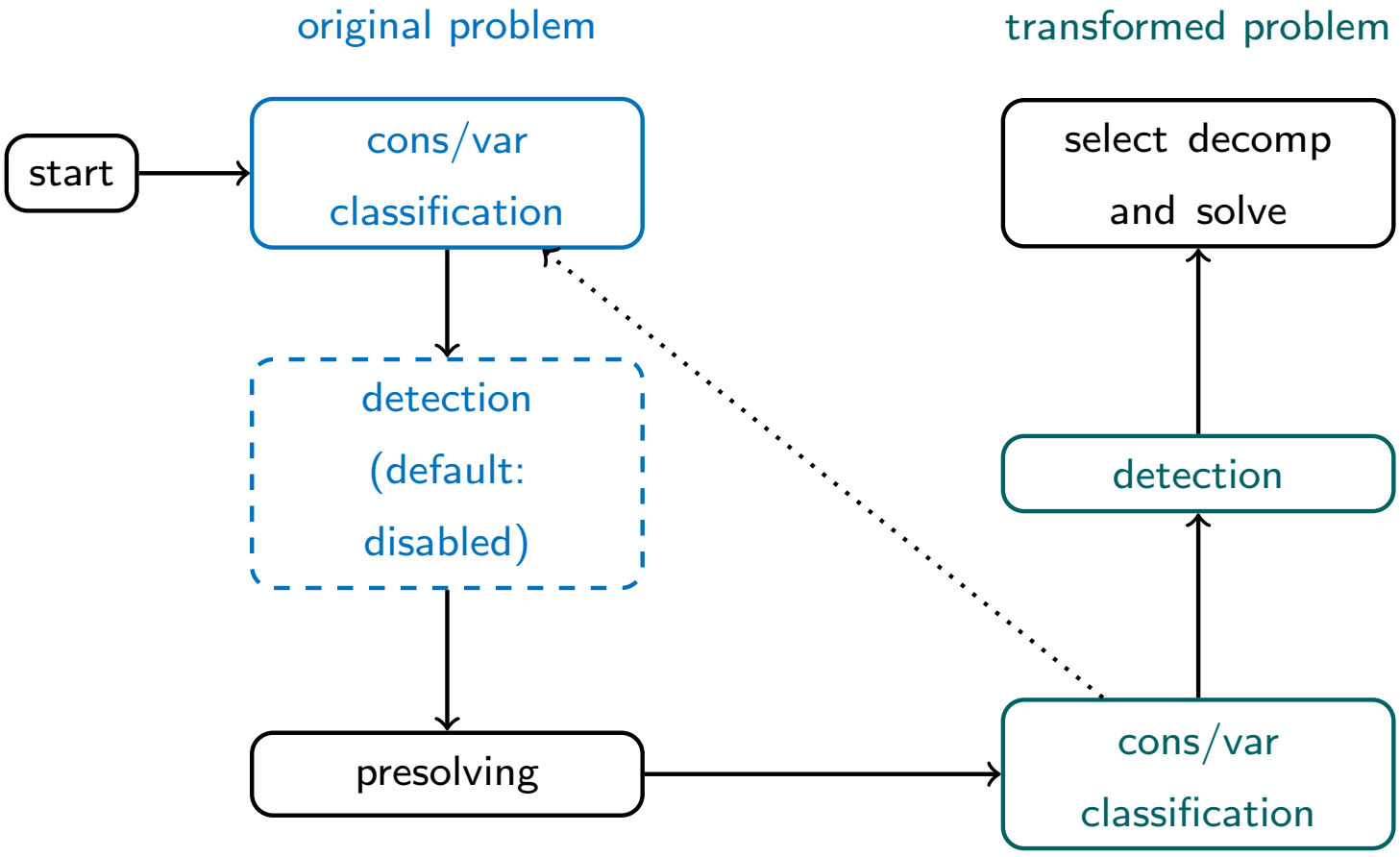
  - ...
- Set every combination of classes as linking vars and master-only variables

Information from original problem can be preserved



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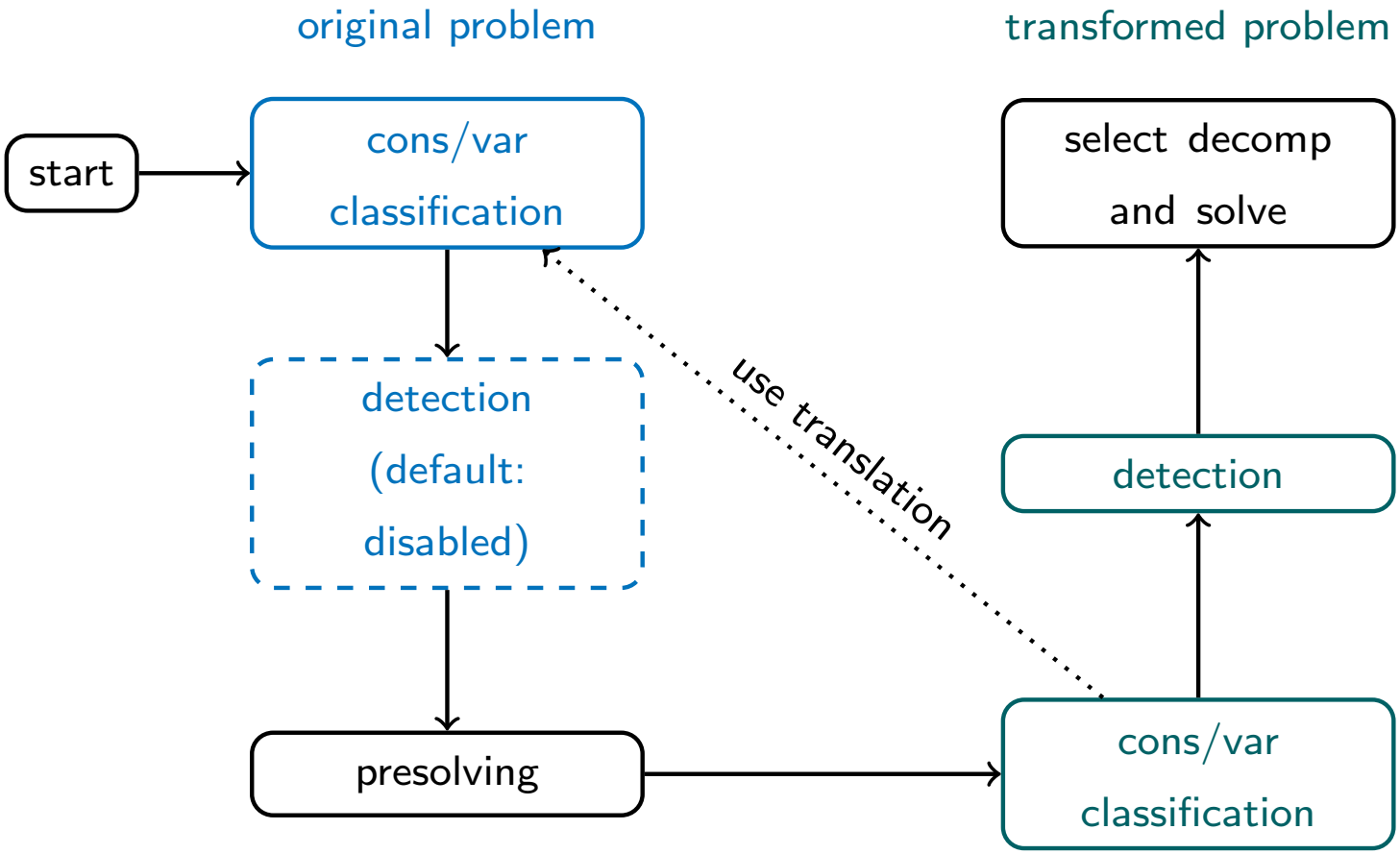

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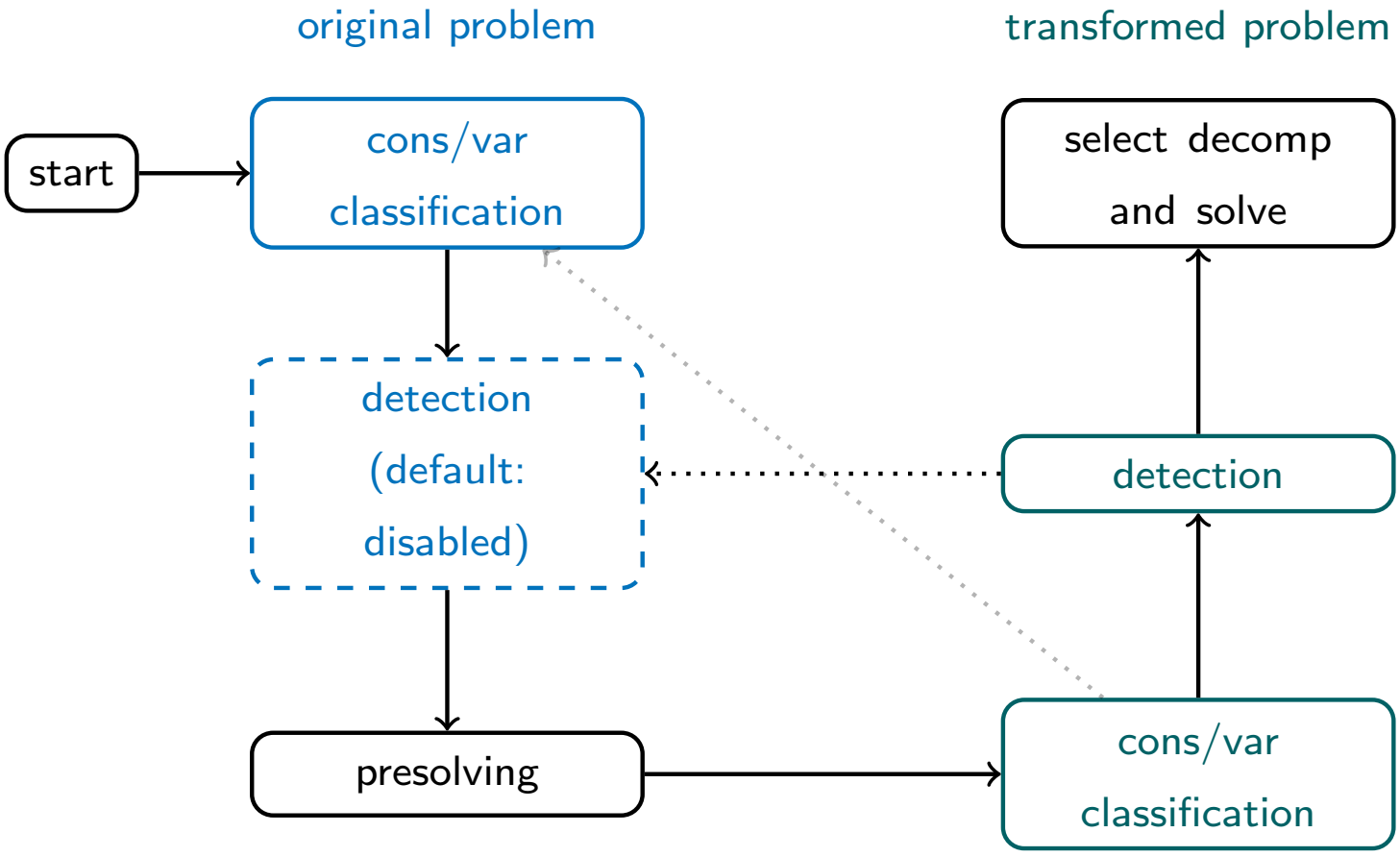

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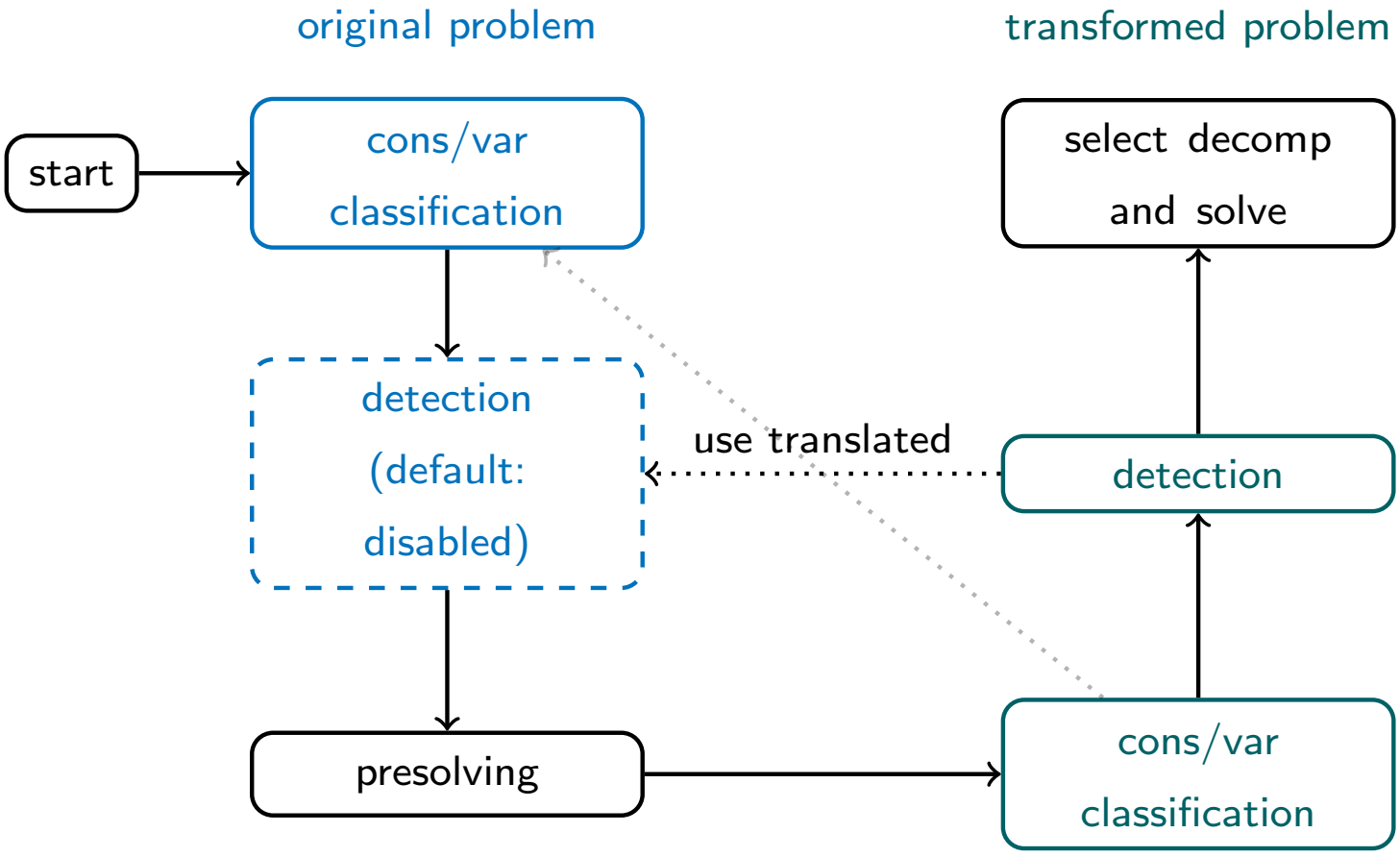

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- originally: CPAIOR13 talk related to Wang and Ralphs (2013) proposed a frequency table/histogram (this is under the assumption that blocks are identical): count how many rows have 1, 2, 3, ... many non-zeros, e.g., for atm**20**-100:

# of Nonzeros	2	11	12	13	24	40	100
# of Rows	2220	**20**	**20**	2	1998	100	**20**

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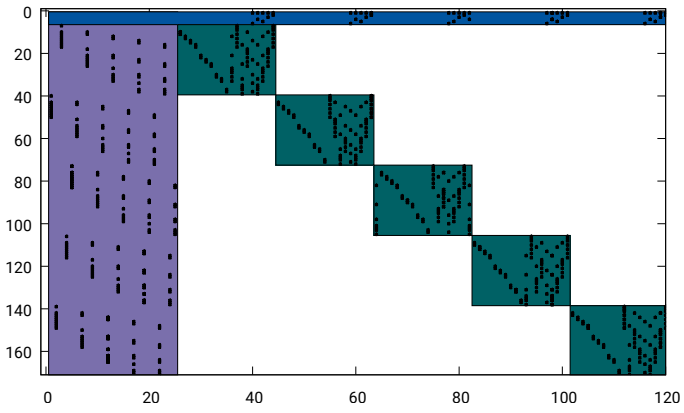
# of Nonzeros	2	11	12	13	24	40	100
# of Rows	2220	**20**	**20**	2	1998	100	**20**

- we calculate greatest common divisors of constraint/variable classes for all classifiers
- thus get a voting for the number of blocks

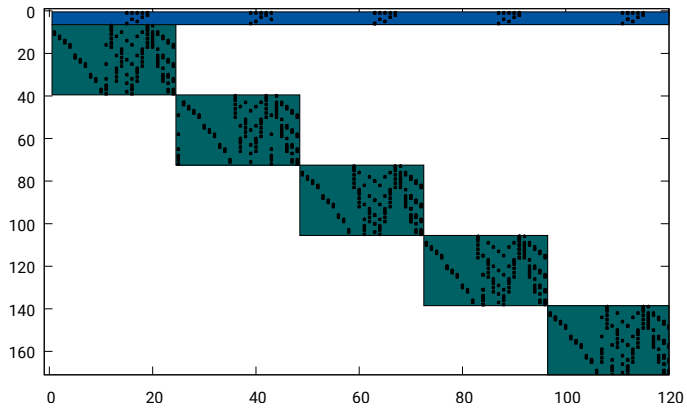
Pluggable scores

score:

- maximize fraction of non-colored area:



score of 0.611



score of 0.77

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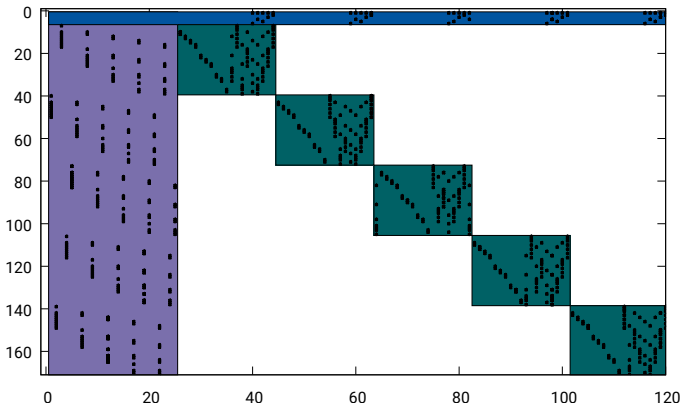

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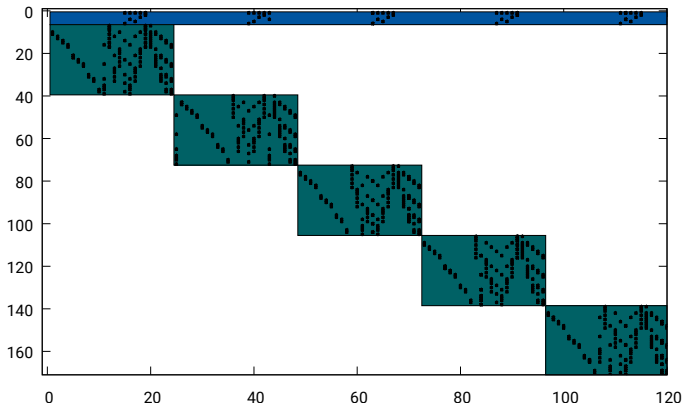


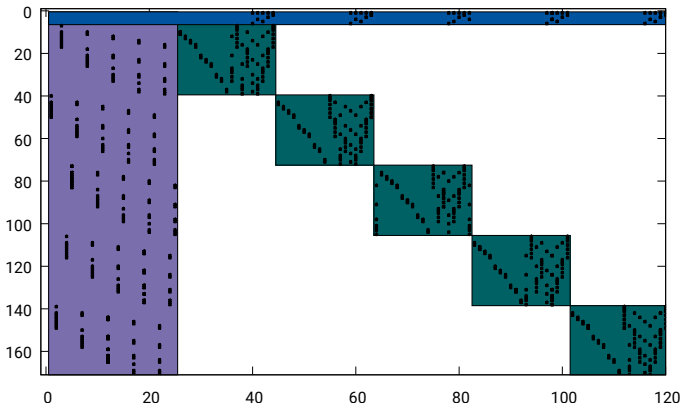
score of 0.77

- currently: 7 different scores, combinations of:
  - consider copied linking vars
  - consider if master consists only of several constraint types
  - consider aggregation information

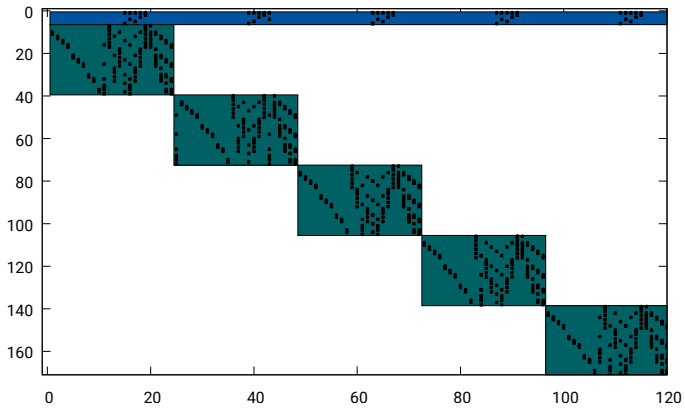
Pluggable scores

score:

- maximize fraction of non-colored area:



score of 0.611



score of 0.77

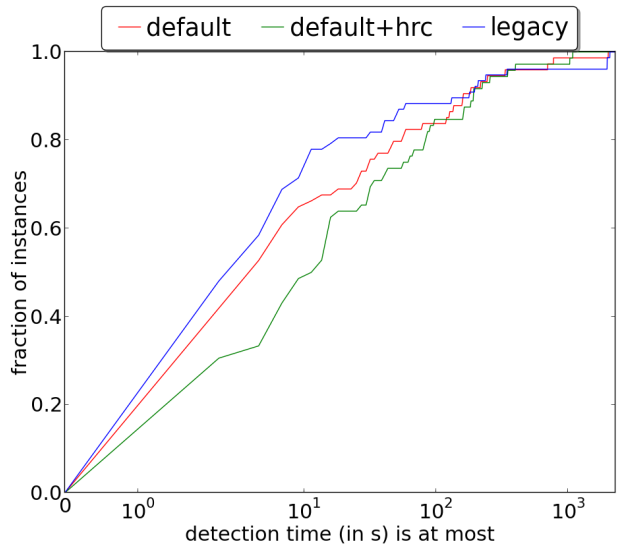
- currently: 7 different scores, combinations of:
  - consider copied linking vars
  - consider if master consists only of several constraint types
  - consider aggregation information
- wip: "strong detection" score (expensive: on demand)

User interface

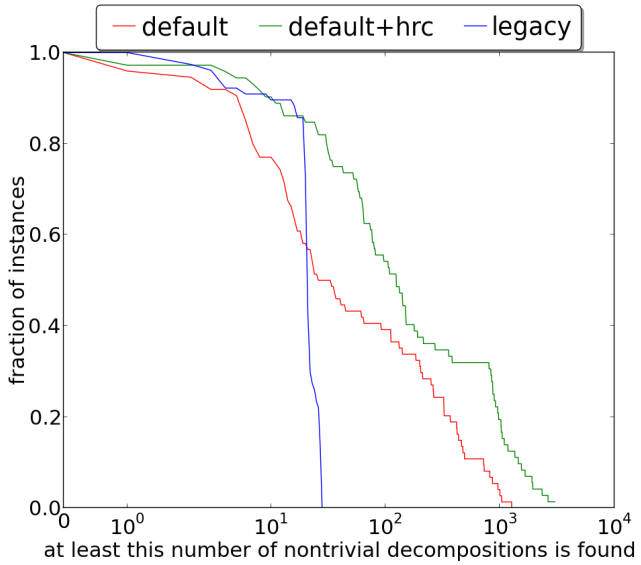
- partial decompositions can be given by user
- toolbox (in development) to call single detectors or assign specific conss or vars by name (regex)
- user can give candidates number of blocks
- browse found decompositions inside gcg and select interesting (for visualization, solving, writing)

Preliminary computational results

- testset: benchmark subset of miplib2010
- three settings: *default*, *default+hrcgpartition*, *legacy*



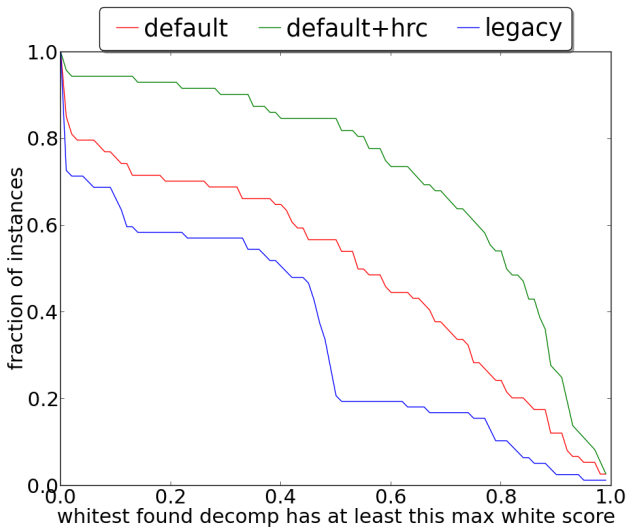
detection time



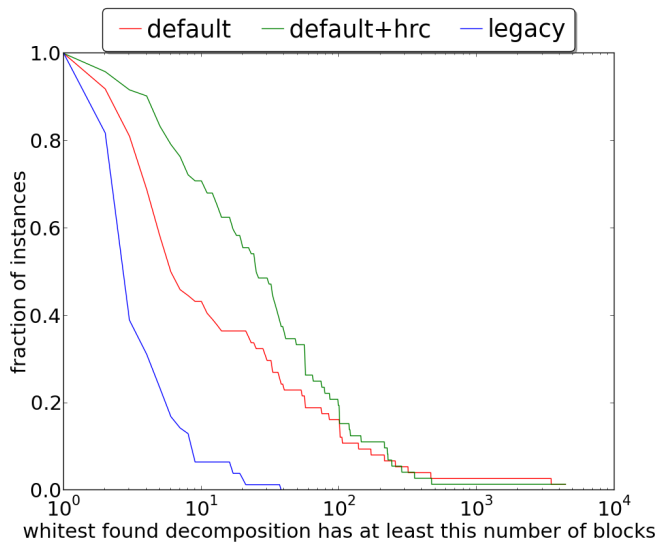
number of found decompositions

Preliminary computational results

- testset: benchmark subset of miplib2010
- three settings: *default*, *default+hrcgpartition*, *legacy*



max white score



nblocks of whitest

Outlook

- tests with termination conditions
- massive tests on resulting reformulations
- test strong decomposition score
- finish toolbox

References

Bergner, M., Caprara, A., Ceselli, A., Furini, F., Lübbecke, M., Malaguti, E., and Traversi, E. (2015). Automatic Dantzig-Wolfe reformulation of mixed integer programs. *Math. Prog.*, 149(1–2):391–424.

Wang, J. and Ralphs, T. (2013). Computational experience with hypergraph-based methods for automatic decomposition in discrete optimization. In Gomes, C. and Sellmann, M., editors, *Integration of AI and OR Techniques in Constraint Programming for Combinatorial Optimization Problems*, pages 394–402. Springer, Berlin, Heidelberg.