A Polyhedral Characterization of Odd Pairs

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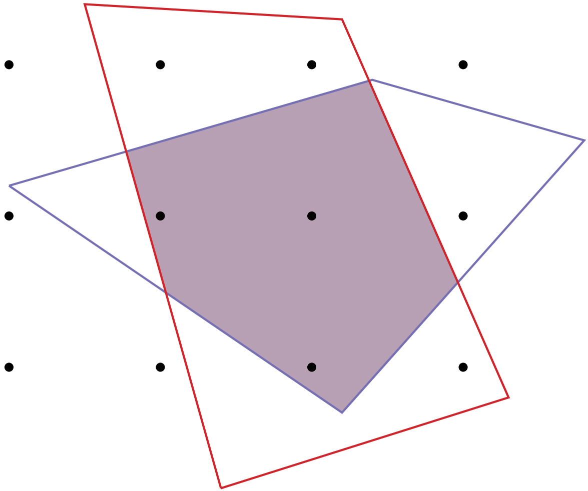
Strength of Dantzig-Wolfe Reformulations



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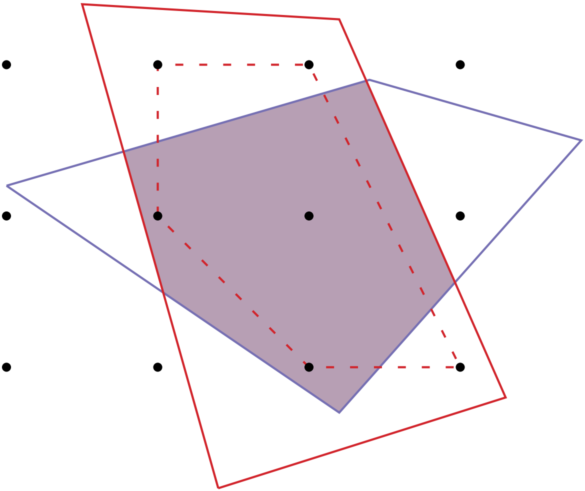

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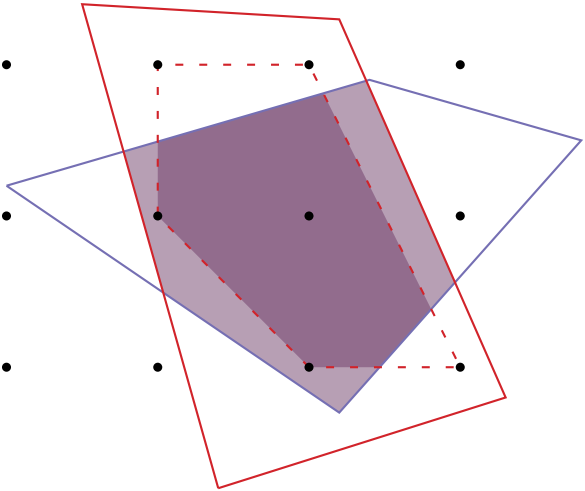

Strength of Dantzig-Wolfe Reformulations: Stable Set

- $G = (V, E), \quad x \in \{0,1\}^V : x_v + x_w \leq 1 \;\; \forall vw \in E$

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Theorem (L., Witt, 2018)

Let $G = (V, E)$ be a graph, $E' \subseteq E$, and $G' := (V, E')$.
The DWR of E' is weakest possible $\iff$ G' is bipartite.

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Let $G = (V, E)$ be a graph, $E' \subseteq E$, and $G' := (V, E')$.
The DWR of E' is strongest possible $\iff$ G' contains all odd induced cycles of G.

Odd Pairs in Graphs

- two vertices v, w in G are an *odd pair* if

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Odd Pairs in Graphs: A Characterization

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Theorem (Witt, L., Reed, 2018)

Let $G = (V, E)$ be a graph and let $v, w \in V$ with $vw \notin E$.

(v, w) is an odd pair $\iff$

$$\mathrm{STAB}(G + vw) = \{x \in \mathrm{STAB}(G) : x_v + x_w \leq 1\} \ .$$

- useful e.g., for iteratively constructing stable set polytopes