Learning how to decompose

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Dantzig-Wolfe Reformulation

$$\begin{array}{ll} \min & cx \\ \text{s.t.} & Ax = b \\ & x \in \mathbb{R}^n_+ \times \mathbb{Z}^p_+ \end{array}$$

- DWR for Mixed Integer Programs
- Solved by column generation

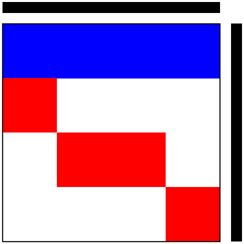


Figure: Decomposition of A

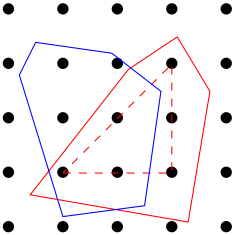


Figure: Tighter relaxation

GCG (Generic Column Generation) Solver

A decomposition: a partition of variables / constraints into blocks

$$
\begin{array}{ll}
\min & c^t x \\
\text{s.t.} & \begin{bmatrix} D^1 & & & & F^1 \\ & D^2 & & & F^2 \\ & & \ddots & & \vdots \\ & & & D^\kappa & F^\kappa \\ A^1 & A^2 & \cdots & A^\kappa & G \end{bmatrix} \cdot \begin{bmatrix} x^1 \\ x^2 \\ \vdots \\ x^\kappa \\ x^\ell \end{bmatrix} \geq \begin{bmatrix} b^1 \\ b^2 \\ \vdots \\ b^\kappa \\ b^\ell \end{bmatrix} \\
& \mathbb{R}^n_+ \times \mathbb{Z}^p_+.
\end{array}
\tag{1}
$$

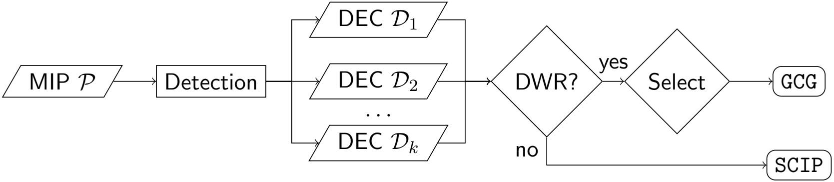
Given:

- a MIP $\mathcal{P}$
- (a decomposition $\mathcal{D}$)

GCG solves (1) using SCIP for:

- Master Problem
- Pricing subproblem

Automatic Decomposition in GCG



A MIP can be forced in several types of decomposition:

- Border
- Staircase
- etc.

GCG performance highly depends on how well the decomposition catches the problem structure.

Our work: a supervised learning approach to *select the best decomposition* (using no decomposition is often the best answer).

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Supervised Learning

Given an *input* X, a *classifier* is a model (function) f that predicts a *variable of interest* $Y \in S$

$$Y = f(X)$$

- f is *binary* classifier if $S = \{0, 1\}$.
- *learn* a classifier: find the f_θ that fits best a *training set* $((x_i, y_i), i = 1, \dots, n)$ among a family $(f_\theta, \theta \in \Theta)$
- Standard binary classifiers / learning algorithms when $X \in \mathbb{R}^d$

Input variables X:

- MIP $\mathcal{P}$
- Decomposition(s) $\mathcal{D}$
- Remaining time t

Variables of interest:

- Should we use `SCIP` or `GCG`?
- Which decomposition should we use?

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Input for Standard Classifiers

We want to use standard classifiers for f (SVM, KNN, etc.). Need an input in $\mathbb{R}^d$.

- Input size not fixed
- Ordering of columns / rows not fixed

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$$\boxed{\begin{array}{c}\text{Input}\\ (\mathcal{P},\mathcal{D},t)\end{array}} \xrightarrow[\text{map } \phi]{\text{Feature}} \boxed{\begin{array}{c}\text{Features vector}\\ \phi(\mathcal{P},\mathcal{D},t)\in\mathbb{R}^d\end{array}} \xrightarrow[f_\theta]{\text{Classifier}} \boxed{\begin{array}{c}\text{Output}\\ f_\theta\left(\phi\left(\mathcal{P},\mathcal{D},t\right)\right)\end{array}}$$

Define ϕ
more than 80 features

Choose f *family*
Learn θ

Features

Examples of features used:

- Time t
- Nb variables/constraints
- Variable types
- Constraint types
- Products of features
- Nb linking variables / constraints
- Nb blocks
- $\min$, $\max$, mean block size
- Detector used (indicator)
- Detection quality metrics

`GCG` decomposition selection tool uses empirical detection quality metrics.

Labeling

Training set

- MIP $\mathcal{P}$
- Decompositions $\mathcal{D}$
- SCIP run on each $\mathcal{P}$
- GCG run on each $(\mathcal{P}, \mathcal{D})$

Given an input $(\mathcal{P}, \mathcal{D}, t)$ we learn a classifier

$$Y = f(\mathcal{P}, \mathcal{D}, t)$$

where $Y = 1$ if GCG on $(\mathcal{P}, \mathcal{D})$ is better than SCIP on $\mathcal{P}$ after t, i.e.

- GCG solves $\mathcal{P}$ and SCIP doesn't
- Both solve $\mathcal{P}$ and GCG is faster
- Neither solve $\mathcal{P}$ but GCG's gap is smaller

Question: Should we use SCIP or GCG?

Classifiers based on Decomposition Quality

$f(\mathcal{P}, \mathcal{D}, t) \in [0,1]$: probability that `GCG` with $\mathcal{D}$ beats `SCIP` after t.

Given the decompositions $\mathcal{D}_1, \ldots, \mathcal{D}_k$ available and the remaining time t, *use GCG if*

$$\max_i f(\mathcal{P}, \mathcal{D}_i, t) \geq \alpha$$

We take $0.5 < \alpha \leq 1$: decomposition is not a default choice.

If we use `GCG`, *select the decomposition*

$$\arg\max_i f(\mathcal{P}, \mathcal{D}_i, t)$$

Question: Which decomposition should we use?

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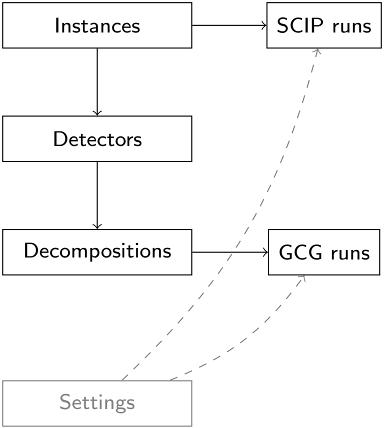
Technical Setup

- database
- python interface
- SCIP version 3.2.1, GCG 2.1.1.
  i7-2600 3.4GHz PC,
  8MB cache, 16GB RAM
- ~ 135 days computing time

Database Schema



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Distribution of Instances

SCIP results	all	structured												non-str
		clr	stcv	cpmp	sdlb	ctst	gap	ntlb	ltsz	bp	rap	stbl	cvrp	miplib
instances	400	25	25	25	25	25	25	25	25	25	25	25	25	100
opt. sol.	65.5%	19	3	18	10	25	23	25	25	6	12	22	6	68
feas. sol.	31.5%	6	21	7	11	-	2	-	-	19	12	3	19	26
no sol.	3.0%	-	1	-	4	-	-	-	-	-	1	-	-	6

Structured Instances

coloring (clr)
set covering (stcv)
capacitated p-median (cpmp)
survivable fixed telecom
network design (sdlb)
cutting stock (ctst)
generalized assignment (gap)
network design (ntlb)
resource allocation (rap)
capacitated vehicle routing (cvrp)
lot sizing (ltsz)
bin packing (bp)
stable set (stbl)

Splitting Training and Testset

Reminder: datapoints $(\mathcal{P}, \mathcal{D}, t)$

Split training and test set by mip instances, to avoid a biased estimator.

Distribution of decompositions per MIP instance:

- Average: ~ 15.3
- Standard Deviation: ~ 9.0

	Instances	Decompositions
Training	269 ($\sim$ 2/3)	4434
Test	131 ($\sim$ 1/3)	2069

Overall Performance

- Test set of 131 MIP instances, 99 structured and 32 unstructured.
- `GCG` better than `SCIP` on 34 instances.

Nearest neighbor classifier of `scikit-learn` library.

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Overall Performance

- Test set of 131 MIP instances, 99 structured and 32 unstructured.
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Nearest neighbor classifier of `scikit-learn` library.

Instances	All				Structured				Non-structured			
Solver	SCIP	GCG	SL	OPT	SCIP	GCG	SL	OPT	SCIP	GCG	SL	OPT
No opt. sol.	52	66	44	39	39	37	31	26	13	29	14	13
CPU time (h)	111.3	142.6	93.1	85.7	83.5	82.2	65.9	58.5	27.8	56.8	29.2	27.2
Geo. mean (s)	127.1	370.4	78.6	67.8	73.4	146.9	39.2	32.2	672.9	5145.0	766.0	646.5

- SCIP: apply SCIP to all instances
- GCG: apply GCG with build-in selection tool
- SL: our supervised learning scheme
- OPT: best decomposition selected each time

Solver Selection Accuracy

Avoid using GCG when there is no appropriate structure.

For $(\mathcal{P}, \mathcal{D}, t)$: Is GCG on $(\mathcal{P}, \mathcal{D})$ better than SCIP on $\mathcal{P}$?

		All instances		Structured		Non-structured	
Classifier	Pred.	SCIP 74.0%	GCG 26.0%	SCIP 68.7%	GCG 31.3%	SCIP 90.6%	GCG 9.4%
RBF Unbal.	SCIP	TN	FN				
	GCG	FP	TP				
KNN distance.	SCIP						
	GCG						
RF Unbal.	SCIP						
	GCG						
RF Bal.	SCIP						
	GCG						

Solver Selection Accuracy

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		All instances		Structured		Non-structured	
		SCIP	GCG	SCIP	GCG	SCIP	GCG
Classifier	Pred.	74.0%	26.0%	68.7%	31.3%	90.6%	9.4%
RBF Unbal.	SCIP	73.3%	19.1%	66.7%	23.2%	90.6%	9.4%
	GCG	3.8%	3.8%	5.1%	5.1%	0.0%	0.0%
KNN distance.	SCIP	69.5%	9.9%	64.6%	11.1%	84.4%	6.3%
	GCG	6.9%	13.7%	7.1%	17.2%	6.3%	3.1%
RF Unbal.	SCIP	63.4%	11.5%	55.6%	13.1%	87.5%	6.3%
	GCG	10.7%	14.5%	13.1%	18.2%	3.1%	3.1%
RF Bal.	SCIP	60.3%	10.7%	50.5%	11.1%	90.6%	9.4%
	GCG	13.7%	15.3%	18.2%	20.2%	0.0%	0.0%

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Best Decomposition Selection Accuracy

Classifier	All instances	GCG selected by class.	GCG the best
RBF	42.7%	80.0%	76.7%
KNN	58.8%	88.9%	77.4%
RF unbalanced	51.1%	72.7%	76.5%
RF balanced	64.9%	71.1%	79.4%

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Best Decomposition Selection Accuracy

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RBF	42.7%	80.0%	76.7%
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How to improve the performance?

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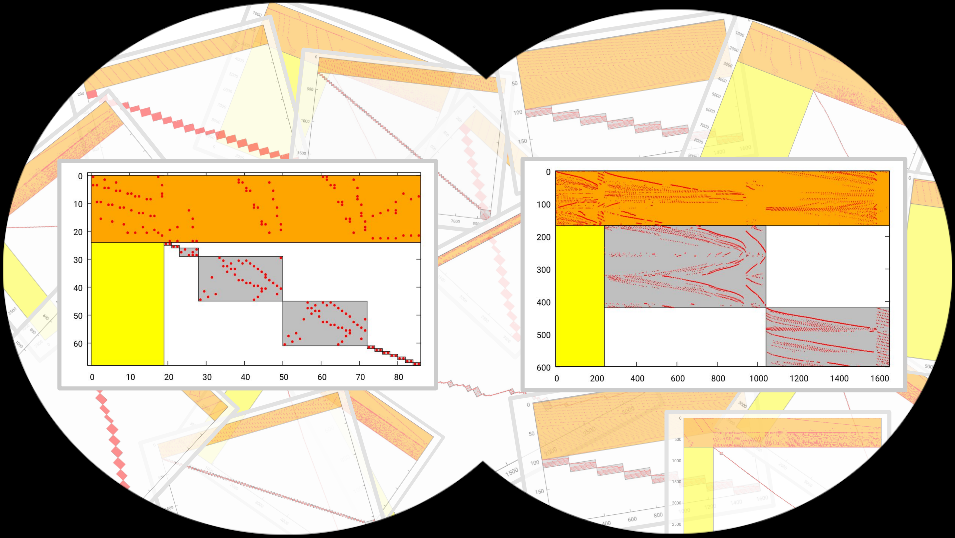
Outlook: a Bunch of Decompositions



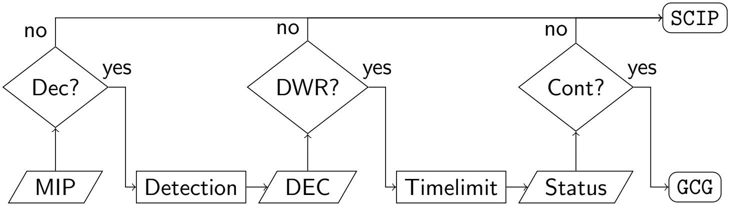
Outlook: a Bunch of Decompositions

- Beat `SCIP` on more instances
- Harder Machine Learning task

Estimation

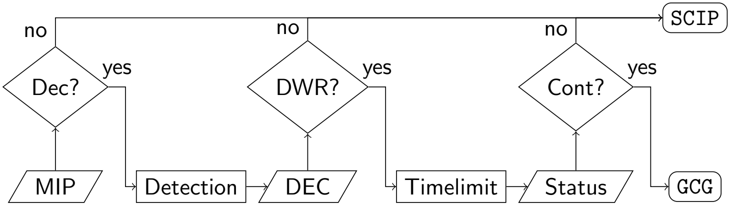
> 1.000 MIPs $\cdot$ 500 DECs/MIP $\cdot$ 2h timelimit ≈ 114 years / #PCs

Additional Decisions in GCG



1. Prediction before detection
2. DWR? and selection
3. Enrich feature quality by exploiting run time features

Additional Decisions in GCG



1. Prediction before detection
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Thank you for your attention!

Backup

- geom. mean: $\bar{x}_{\mathsf{geom}} = \sqrt[n]{\prod_{i=1}^{n} x_i}$
- un/balanced: weights associated with classes.