Resource constrained shortest path algorithm for EDF short-term thermal production planning problem

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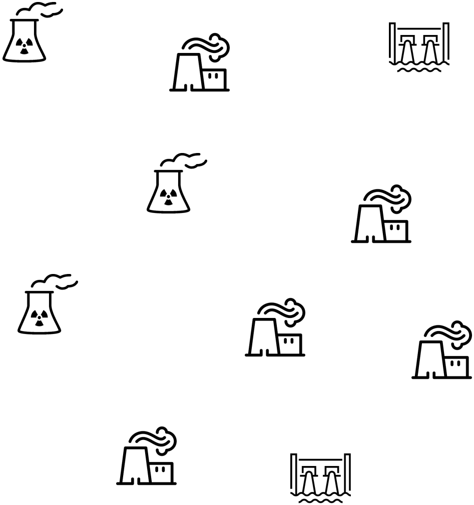
[...] EDF short-term thermal production planning problem



Generation units

- ~ 60 nuclear
- ~ 100 thermal
- ~ 500 hydraulic

Technically feasible production schedules

Min operating cost

Horizon: $24\text{h} + 24\text{h}$

Time limit: 15min

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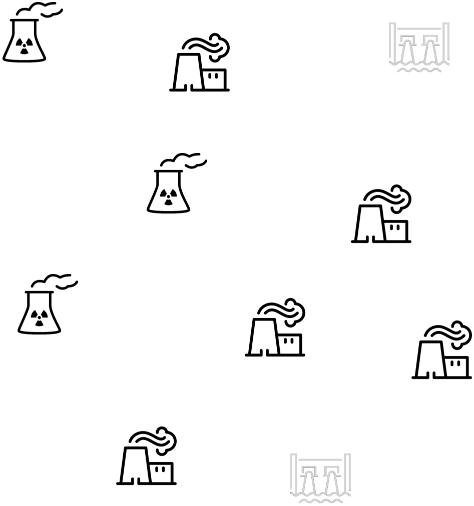

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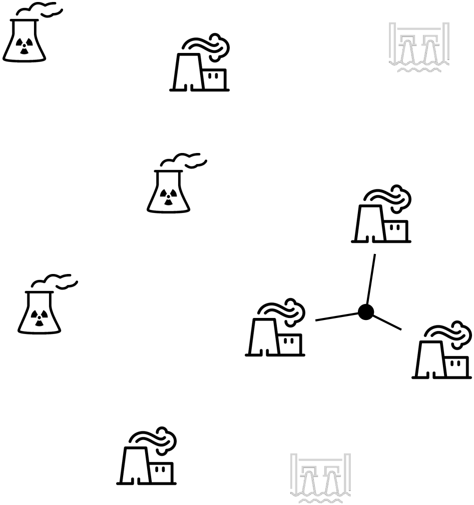

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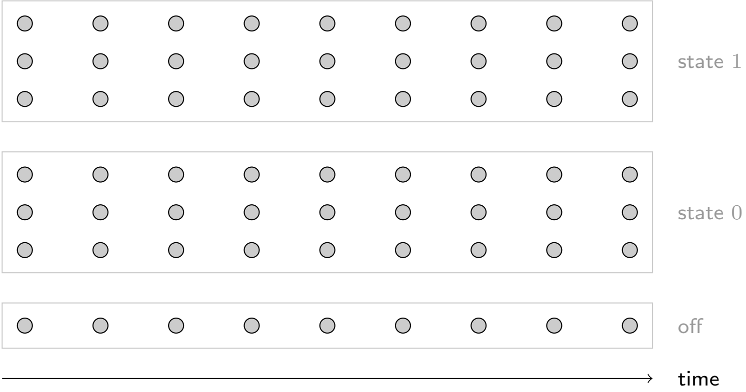

Resource constrained shortest path algorithm [...]

startup · #startups

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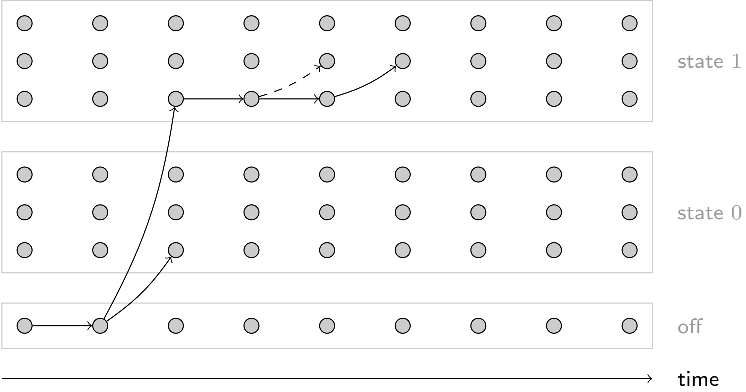

startup · #startups · min_duration_production_level

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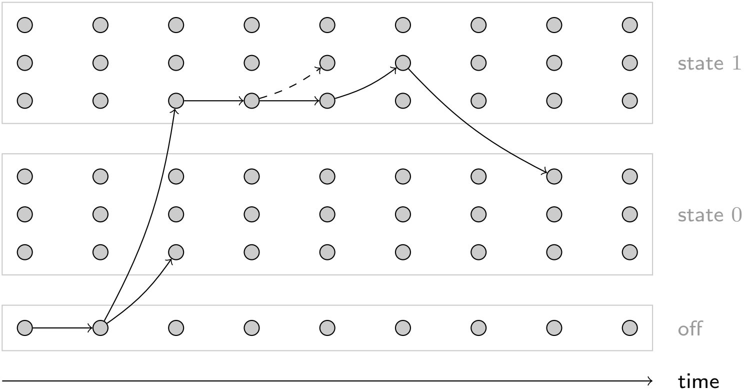

startup · #startups · min_duration_production_level
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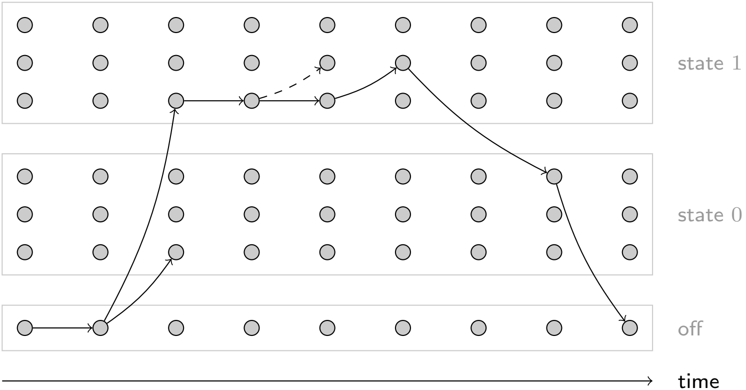

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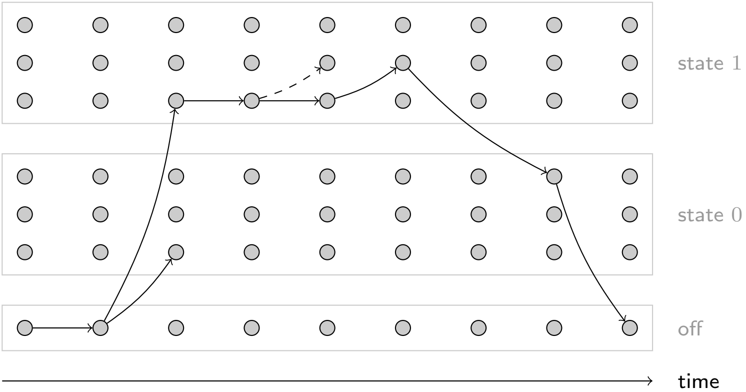

startup · #startups · min_duration_production_level
min_duration_power_state · #modulations · shutdown
min/max_power_state · min/max_increase/decrease
#deep_decreases

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Introduction of the Unit Commitment Problem

Part of lagrangian relaxation approach:

Input:

- discretization of the time horizon (96 steps)
- operation cost of the plants depending on power level
- electricity prices for each time step
- technical constraints

Output:

- production plan
- such that: technical constraints are respected
- which: maximizes profit - operating cost

Model & Scope

Assume we can generate all technically feasible production plans $\mathcal{P}_i$ for each plant $i \in V$.

$$\begin{array}{rrcll} \text{minimize} & \sum_{p \in \mathcal{P}} c_p x_p & & & \\ \text{subject to} & \sum_{p \in \mathcal{P}_i} x_p & = & 1 & \forall i \in V \\ & x_p & \in & \{0,1\} & \forall p \in \mathcal{P} \end{array}$$

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\begin{aligned}
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Scope of this talk

- assume problems are independent
- solve each subproblem separately

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Scope of this talk

- assume problems are independent
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Problem faced in reality

- linked by shared gas stock
- column generation approach
- pricing is still RCSPP

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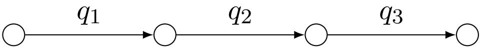
Shortest Path in an Ordered Monoid

For each arc a a resource $q_a \in \mathcal{R}$

- Associative binary operator $\oplus$: path resources
- Neutral element 0: empty path



$$q_P = q_1 \oplus q_2 \oplus q_3$$

$(\mathcal{R}, \oplus)$ is a monoid.

- An order $\preceq$ compatible with $\oplus$: $\quad q \preceq \tilde{q} \Rightarrow \begin{cases} r \oplus q \preceq r \oplus \tilde{q} \\ q \oplus r \preceq \tilde{q} \oplus r \end{cases}$

$(\mathcal{R}, \oplus, \preceq)$ is an ordered monoid.

- Non-decreasing cost c and constraint ρ functions.

Shortest Path with Resources in an Ordered Monoid

Given an ordered monoid $(\mathcal{R}, \oplus, \preceq)$

Input:

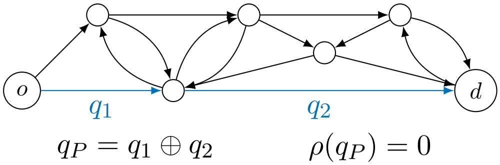
- Digraph $D = (V, A)$
- Two vertices $o, d \in V$
- Resources $q_a \in \mathcal{R}$
- Two non-decreasing oracles $c : \mathcal{R} \to \mathbb{R}$ $\rho : \mathcal{R} \to \{0, 1\}$

Output:

- An o-d path P such that
  $$\rho\left(\bigoplus_{a \in P} q_a\right) = 0$$
  which minimizes
  $$c\left(\bigoplus_{a \in P} q_a\right)$$

Cost and constraint(s):

- non-linear(s)



Operations Research

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Example 1: Usual Resource Constrained Shortest Path

Input:

- Digraph $D = (V, A)$
- Origin o, Destination d
- Costs $c_a \in \mathbb{R}$
- Weights $w_a^i \in \mathbb{R}$ for $i \in [n]$
- Thresholds $W^i \in \mathbb{R}$ for $i \in [n]$

Output:

- An o-d path P such that
$$\sum_{a \in P} w_a^i \leq W^i \quad \forall i \in [n]$$
which minimizes $\sum_{a \in P} c_a$

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Model:

- $\mathcal{R} = \mathbb{R}^{n+1}$
- $q_a = (c_a, w_a^1, \ldots, w_a^n)$
- $c : ((q^0, \ldots, q^n)) \mapsto q^0$
- $\rho : ((q^0, \ldots, q^n)) \mapsto \max_{i \in \{1,\ldots,n\}} \mathbb{1}_{q^i > W^i}$

Example 2: Restricting Startups

Input:

- Digraph $D = (V, A)$
- Origin o, destination d
- $w_a = \begin{cases} 1, & \text{if startup arc,} \\ 0, & \text{otherwise.} \end{cases}$
- Max startups W^{start}

Output:

- An o-d path P of minimum cost such that the number of startups per plant is not greater than W^{start}.

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- Max startups W^{start}

Output:

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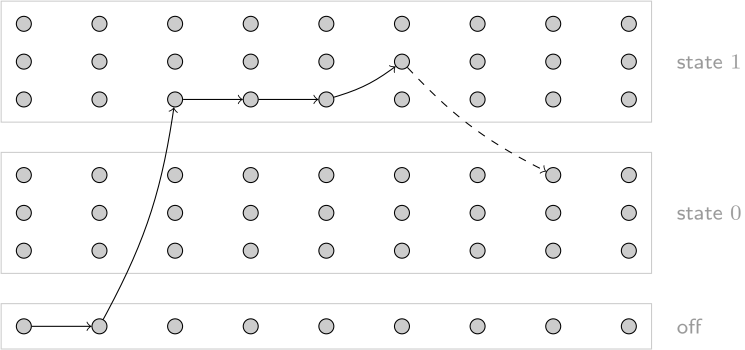
Model:

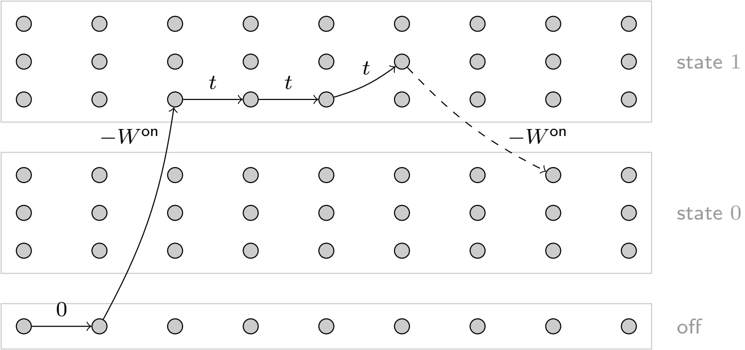
- $\mathcal{R} = \mathbb{R}^2$
- $q_a = (c_a, w_a)$
- $c : ((q^0, q^1)) \mapsto q^0$
- $\rho : ((q^0, q^1) \mapsto \mathbb{1}_{q^1 > W^{\text{start}}}$

Example 3: Minimum Duration in Online State



Stay in online state for at least W^{on}.

Example 3: Minimum Duration in Online State



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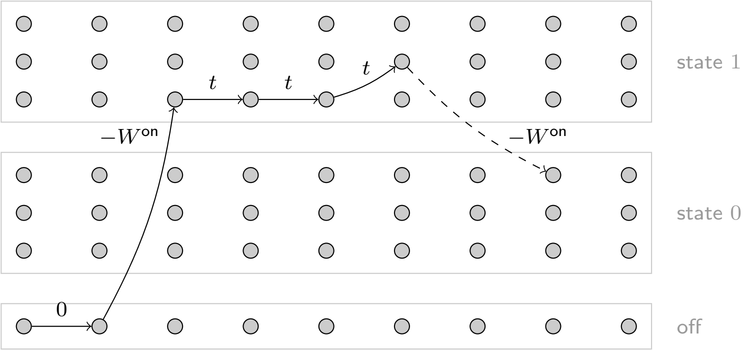

Stay in online state for at least W^{on}.

$$(c_a, w_a^1) \oplus (c_{a'}, w_{a'}^1) = \begin{cases} \infty & \text{, if } w_a^1 < 0 \land w_{a'}^1 < 0, \\ w_{a'}^1 & \text{, if } w_a^1 \geq 0 \land w_{a'}^1 < 0, \\ w_a^1 + w_{a'}^1 & \text{, otherwise} \end{cases}$$

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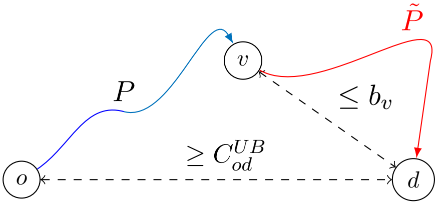
Usual A* algorithm



- $q_P \in \mathbb{R}$
- $C_{od}^{UB} \geq \min_{P \in \mathcal{P}_{o,d}} q_P$
- $b_v \leq q_P, \forall P \in \mathcal{P}_{vd}$

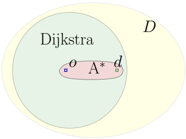
A path $P \in \mathcal{P}_{ov}$ satisfying $q_P + b_v > C_{od}^{UB}$ is not the subpath of an optimal path.

A* algorithm: a Branch & Bound

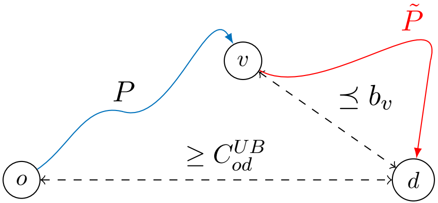
- Generate all the paths satisfying

$$q_P + b_v \leq C_{od}^{UB}$$

- Update C_{od}^{UB}



Generalized A* algorithm



- $q_P \in \mathcal{R}$
- $C_{od}^{UB} \geq \min_{P|\rho(P)=0} c(q_P)$
- $b_v \preceq q_{\tilde{P}}, \forall \tilde{P} \in \mathcal{P}_{vd}$

A path $P \in \mathcal{P}_{ov}$ satisfying $c(q_P \oplus b_v) > C_{od}^{UB}$ or $\rho(q_P \oplus b_v) = 1$ is not the subpath of an optimal path.

Generalized A* Algorithm: a Branch & Bound

- Generate all the paths satisfying

$$c(q_P \oplus b_v) \leq C_{od}^{UB} \quad \text{and} \quad \rho(q_P \oplus b_v) = 0 \qquad \text{(Low)}$$

- Update C_{od}^{UB}

Generic enumeration algorithm

Preprocessing.

$L \leftarrow$ empty path in o

$c_{od}^{UB} \leftarrow +\infty$.

While L is not empty:

- Extract from L a path P of minimum key.
- If $v = d$ and $\rho(P) = 0$, $c_{od}^{UB} \leftarrow \min(c_{od}^{UB}, c(P))$.
- Test if P must be extended. If yes:
  - for each $a \in \delta^+(v)$, add $P + a$ to L.

L: cand. paths list.
c_{od}^{UB}: Upper bound on optimal path cost.
v: destination of P.

Add. structures

b_v: lower bound on v-d paths q_P
M_v: list of non dominated o-v paths

Algorithm	Test	Key
Generalized A*	(Low)	$c(q_P \oplus b_v)$
Label dominance	(Dom)	$c(q_P)$
Label correcting	(Dom), (Low)	$c(q_P \oplus b_v)$

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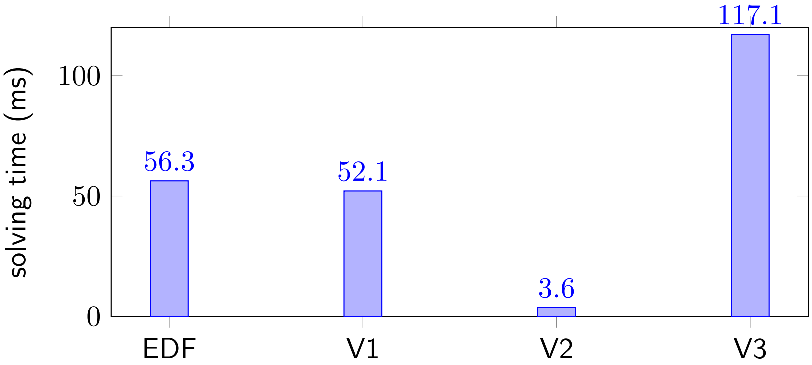
Some Numbers

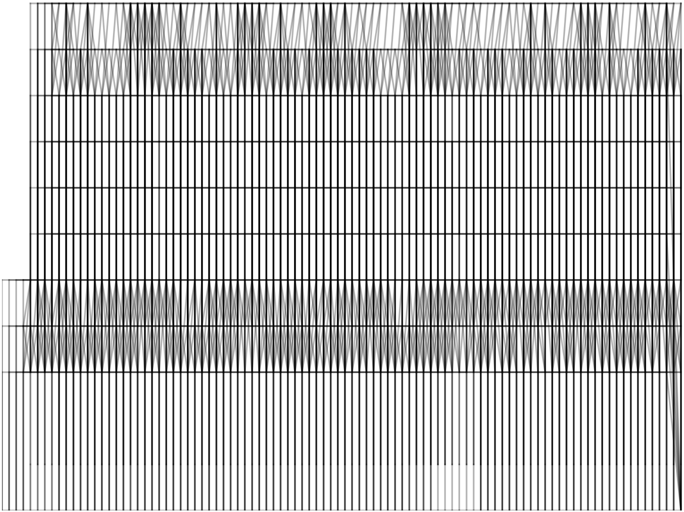
- Plants
  - 8 gas plants $(3, 3, 2)$
  - 97 non-gas plants
- Size of graph (depends on the model)
  - ~ 2.000 nodes
  - ~ 10.000 arcs

Solving Non-Gas Subproblem

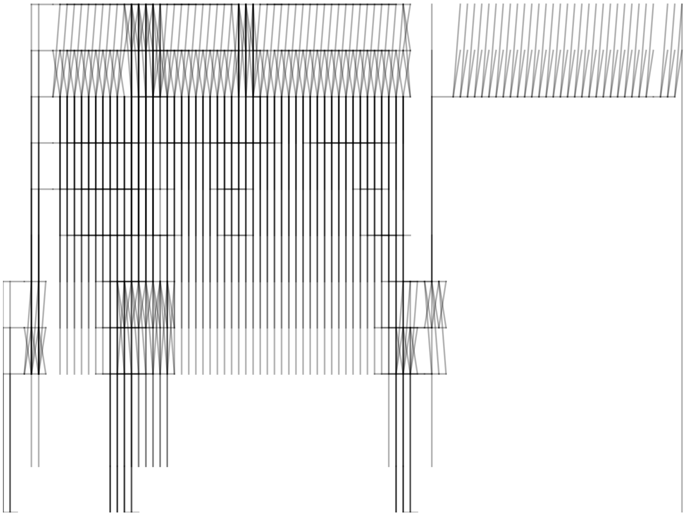


selection	node	node	path	path
key	early date	early date	$q_p \oplus b_v$	q_p
#iter	2.100	1.914	3.596	230.763
#dis dom	190k	137k	9k	201k
#dis bound	0	79k	9k	1.029k
#od paths	327	14	1	899
speedup	1.00x	0.93x	0.06x	2.08x

Arcs touched by previous algorithm



Arcs touched by our algorithm



Summary

- Redesign of the graph
- Modeling unit commitment problem as RCSPP in lattice ordered monoid
- (Conditional) bound computation
- Using a Generalized A* algorithm
- Solving of nongas / gas pricing problems
- Column generation - Multi Unit Commitment Problem
- Pareto Frontier